Pearson Edexcel

Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

May/June 2023 Exemplar
9MA0-02 A level Mathematics
Paper 02: Pure Mathematics

Senior Examiner’s feedback on student responses

Contents

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About this booklet

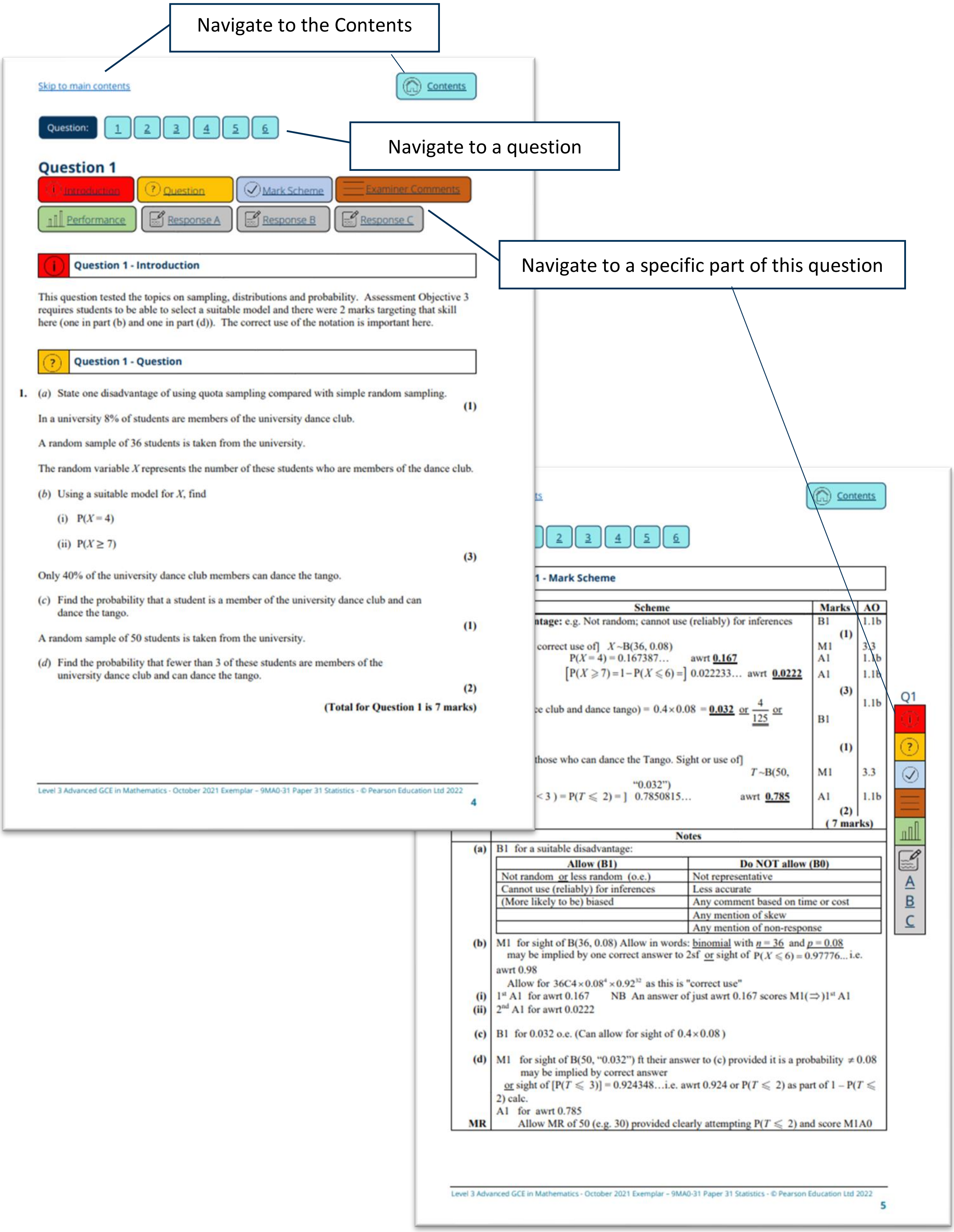
This document has been produced to support mathematics teachers delivering the GCE Mathematics specification.

This document looks at questions from the 9MA0-02 A level Mathematics Paper 2: Pure Mathematics June 2023 examination paper. It shows real student responses to these questions, and how the examining team follow the mark schemes to demonstrate how the students are awarded the marks. For the mark scheme notes and details of alternative methods please see the full mark scheme for this question paper on our website.

For 2023, the approach all exam boards have taken to grading was to return to pre-pandemic grading, while giving students protection against any impact of disruption. Results in summer 2023 therefore will be far more in line with summer exams that were sat in 2019, but lower than in 2022, when grades awarded were based on a mid-point between 2019 and 2021 outcomes. For more information please read our 'Understanding grade boundaries 2023' document.

* The question level performance data is there to give an indication only of how students performed, on each question, in the context of sitting the entire exam paper and is not an indication of how students may perform sitting a question in isolation. The performance data of this series doesn't represent a normal series due to small number of entries.

How to use this booklet



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Question 1 - Introduction

This question tested the topics on sampling, distributions and probability. Assessment Objective 3 requires students to be able to select a suitable model and there were 2 marks targeting that skill here (one in part (b) and one in part (d)). The correct use of the notation is important here.

Question 1 - Question

1. (*a*) State one disadvantage of using quota sampling compared with simple random sampling. **(1)**

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(*b*) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geq 7)$ **(3)**

Only 40% of the university dance club members can dance the tango.

(*c*) Find the probability that a student is a member of the university dance club and can dance the tango. **(1)**

A random sample of 50 students is taken from the university.

(*d*) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango. **(2)**

(Total for Question 1 is 7 marks)

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Contents

2 3 4 5 6

1 - Mark Scheme

Scheme	Marks	AO
ntage: e.g. Not random; cannot use (reliably) for inferences	B1 (1)	1.1b
correct use of] $X \sim B(36, 0.08)$	M1	3.3
$P(X = 4) = 0.167387...$ awrt **0.167**	A1	1.1b
$[P(X \geqslant 7) = 1 - P(X \leqslant 6) =]$ $0.022233...$ awrt **0.0222**	A1 (3)	1.1b
e club and dance tango) $= 0.4 \times 0.08 =$ **0.032** or $\frac{4}{125}$ or	B1 (1)	1.1b
those who can dance the Tango. Sight or use of] $T \sim B(50,$ "0.032")	M1	3.3
$< 3) = P(T \leqslant 2) =]$ $0.7850815...$ awrt **0.785**	A1 (2)	1.1b
	(7 marks)	

Notes

(a) B1 for a suitable disadvantage:

Allow (B1)	Do NOT allow (B0)
Not random or less random (o.e.)	Not representative
Cannot use (reliably) for inferences	Less accurate
(More likely to be) biased	Any comment based on time or cost
	Any mention of skew
	Any mention of non-response

(b) M1 for sight of B(36, 0.08) Allow in words: binomial with $n = 36$ and $p = 0.08$ may be implied by one correct answer to 2sf or sight of $P(X \leqslant 6) = 0.97776...$ i.e. awrt 0.98

Allow for $36C4 \times 0.08^4 \times 0.92^{32}$ as this is "correct use"

(i) 1st A1 for awrt 0.167 NB An answer of just awrt 0.167 scores M1(⇒)1st A1

(ii) 2nd A1 for awrt 0.0222

(c) B1 for 0.032 o.e. (Can allow for sight of 0.4×0.08)

(d) M1 for sight of B(50, "0.032") ft their answer to (c) provided it is a probability $\neq 0.08$ may be implied by correct answer or sight of $[P(T \leqslant 3)] = 0.924348...$ i.e. awrt 0.924 or $P(T \leqslant 2)$ as part of $1 - P(T \leqslant 2)$ calc.

A1 for awrt 0.785

MR Allow MR of 50 (e.g. 30) provided clearly attempting $P(T \leqslant 2)$ and score M1A0

Q1 A B C

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General Examiner Feedback

This paper offered plenty of opportunity for students to show what they had learnt. The early questions were accessible to candidates of all abilities and this was reflected by candidates regularly scoring full marks on several of the early questions. The longer, later questions provided suitable challenge for stronger candidates but also gave opportunities for restarts for students who struggled with earlier parts of questions.

There were particular instances where marks were lost unnecessarily and the following observations should be helpful to students:

- answers are sometimes not given in the form required, e.g. straight line equations such as in questions 4(b) and 9(b)
- values are sometimes not given as exact or to the required accuracy as demanded in the question, e.g. the exact value of R and the value of α to 3 decimal places in 8(a)
- answers not written as printed on the question paper e.g. the "= 0" missing from 3(a)
- answers not given as requested, e.g. the complete equation not given in 4(b) and the partial fractions not written down in 10(a)

These points aside, there were many succinct and elegant solutions to the more demanding problems towards the end of the paper.

Question 1

Question 1 - Question

1.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(*a*) Find $f''(x)$

(2)

(*b*) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

(Total for Question 1 is 4 marks)

Question 1 - Mark Scheme

Question	Scheme	Marks	AOs
1(a)	$\{f'(x)=\}...x^2+...x+...\Rightarrow\{f''(x)=\}...x+...$	M1	1.1b
	$\{f'(x)=\}3x^2+4x-8\Rightarrow\{f''(x)=\}6x+4$	A1cso	1.1b
		(2)	
(b)(i)	$"6x+4"=0\Rightarrow x="-\frac{2}{3}"$	B1ft	1.1b
(ii)	$x\leqslant"-\frac{2}{3}"$ or $x<"-\frac{2}{3}"$	B1ft	2.2a
		(2)	
			(4 marks)

Question 1 - Examiner Comments

This question provided an accessible start to the paper for the vast majority of candidates, most of whom picked up at least some of the marks available.

In part (a) most candidates were able to differentiate f (x) twice and were able to earn both marks. If candidates did lose marks it was mainly due to arithmetic or processing errors such as differentiating $3x^2$ to $5x$. The most common error was a slip in the final coefficient of x.

In (b) part (i), most candidates were able to make a successful attempt to solve f $''(x) = 0$ for x. The mark in this part was sometimes lost due to sign errors when rearranging. It was surprisingly common to see $6x + 4 = 0$ leading to $x = \frac{2}{3}$. A significant number of candidates did not simplify their answer, but this was condoned as were values which followed correctly from an incorrect second derivative of the correct form.

Part (ii) proved to more of a challenge and many responses highlighted a lack of understanding or awareness of the condition for a function to be concave. A number of blank responses were seen here. Many who attempted this part of the question deduced, or guessed, that it was necessary to consider the sign of the second derivative but were often confused about which sign was required. Others attempted incorrect calculations setting f $'(x) = 0$ and solving the resulting quadratic in x. Occasionally, candidates attempted sketches of f (x) and/or f $'(x)$ in order to determine the range of f (x) which was not required. Others attempted to demonstrate a sign change in attempt to prove that the solution from part (i) was an inflection point. For those employing the correct method, both strict and non-strict inequalities were accepted.

Q1

?

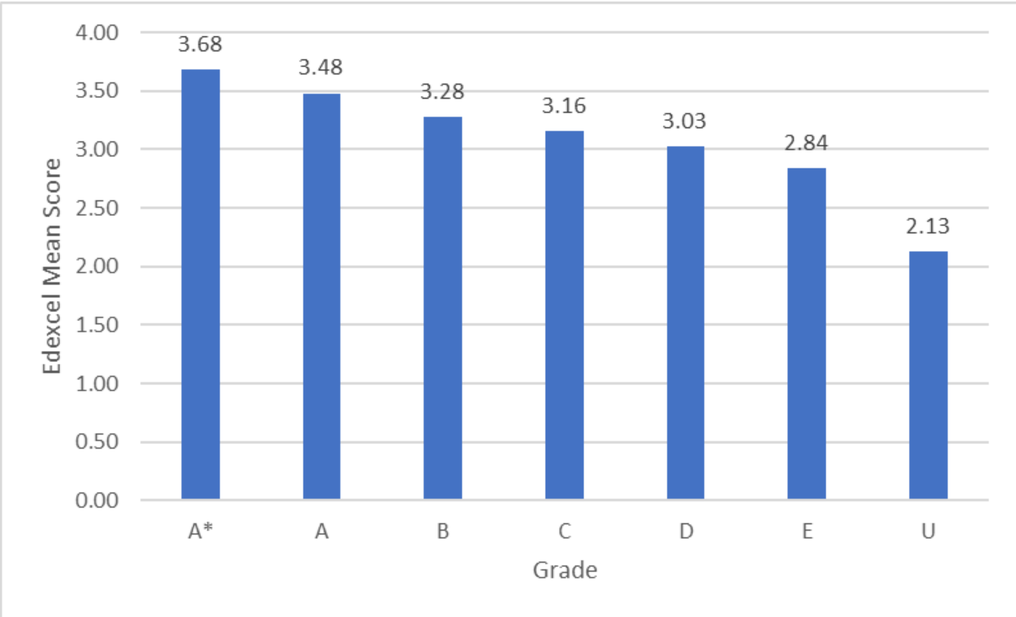
A

B

C

Question 1 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
3.25	4	81%	3.68	3.48	3.28	3.16	3.03	2.84	2.13



Q1
?
A
B
C

Question 1 - Response A

a) $x^3 + 2x^2 - 8x + 5$

$\frac{dy}{dx}$ $3x^2 + 2x - 8$

$f''(x) = 6x + 2$

f'' means differentiate twice

b) $6x + 2 = 0$

$6x = 2$

$x = \frac{2}{6}$

$x = \frac{1}{3}$

1/4 marks

Part (a)
M1: Scored for any of the 3 conditions specified in the mark scheme
A0: Incorrect second derivative

Part (b)(i)

B0ft: This mark was available but does not follow their equation. They should have e.g. $x = -\frac{2}{6}$

Part (b)(ii)
B0ft: No attempt

Question 1 - Response B

a) $f(x) = x^3 + 2x^2 - 8x + 5$
$f'x = 2x^2 + 4x - 8$
$f''x = 4x + 4$

b) i $4x + 4 = 0$
$4x = 4 \quad x = 1$

ii is concave when $f'(x) < 0$
$4x + 4 < 0$
$x < 1$

2/4 marks

Part (a)
M1: Scored for any of the 3 conditions specified in the mark scheme.
A0: Incorrect second derivative.

Part (b)(i)
B0ft: Incorrect follow through value from their $ax + b = 0$. They should have $x = -1$

Part (b)(ii)
B1ft: Follows their value of x which was obtained from an equation of the required form.

Examiner comments
Note that the mark scheme allows follow through for both marks in part (b).

Question 1 - Response C

a) $f'(x) = 3x^2 + 4x - 8$

$f''(x) = 6x + 4$

bi) $6x + 4 = 0$

$6x = -4$

$x = -\frac{4}{6}$

concave ≤ 0.

bii) concave for $x \leq -\frac{4}{6}$.

4/4 marks

Part (a)
M1: Scored for any of the 3 conditions specified in the mark scheme.
A1: Correct second derivative.

Part (b)(i)
B1: Correct value. Note that an unsimplified value was allowed.

Part (b)(ii)
B1: Correct inequality. Note that strict or non-strict inequalities were allowed here.

Question 2

Question 2 - Question

2. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(*a*) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(*b*) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

(Total for Question 2 is 6 marks)

Question 2 - Mark Scheme

Question	Scheme	Marks	AOs
2(a)(i)	e.g. $(u_2 =)35+7\cos\left(\frac{\pi}{2}\right)-5(-1)^1=40$ *	B1*	2.1
(ii)	$u_3=40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2\ (=28)$ **or** $u_4="28"+7\cos\left(\frac{3\pi}{2}\right)-5(-1)^3\ (=33)$	M1	1.1b
	$u_3=28$ and $u_4=33$	A1	1.1b
		(3)	
(b)(i)	$(u_5=)35$	B1	2.2a
(ii)	e.g. $\sum_{r=1}^{25}u_r=6(35+40+"28"+"33")+35$	M1	3.1a
	$=851$	A1	1.1b
		(3)	
		(6 marks)	

Question 2 - Examiner Comments

For the most part candidates answered this question well. This was dependent on whether they correctly understood the iteration formula.

In part (a) (i), although many scored this first mark, there was a common misunderstanding of how a recurrence relationship works with confusion as to the meaning of n. Rather than term position, it was often confused with u_n so that the incorrect calculation $u_2=35+7\cos\left(\frac{35\pi}{2}\right)-5(-1)^{35}$ was often seen, giving the correct answer but obviously not scoring the B mark. Also not uncommonly seen was n replaced by u_{n+1}, with candidates stating $u_2=40+7\cos\left(\frac{40\pi}{2}\right)-5(-1)^{40}$.

The errors stated above also affected the performance of some candidates in part (ii). Furthermore, a minority of candidates had difficulty with accuracy in the sign of the final term, struggling with the odd/even power of -1. For example, when calculating u_3, $u_3=40+7\cos\left(\frac{2\pi}{2}\right)-5(-1)^2=40-7+5=38$, was not uncommon. Occasionally, candidates were seen to be working with degrees rather than radians, also leading to loss of the accuracy mark.

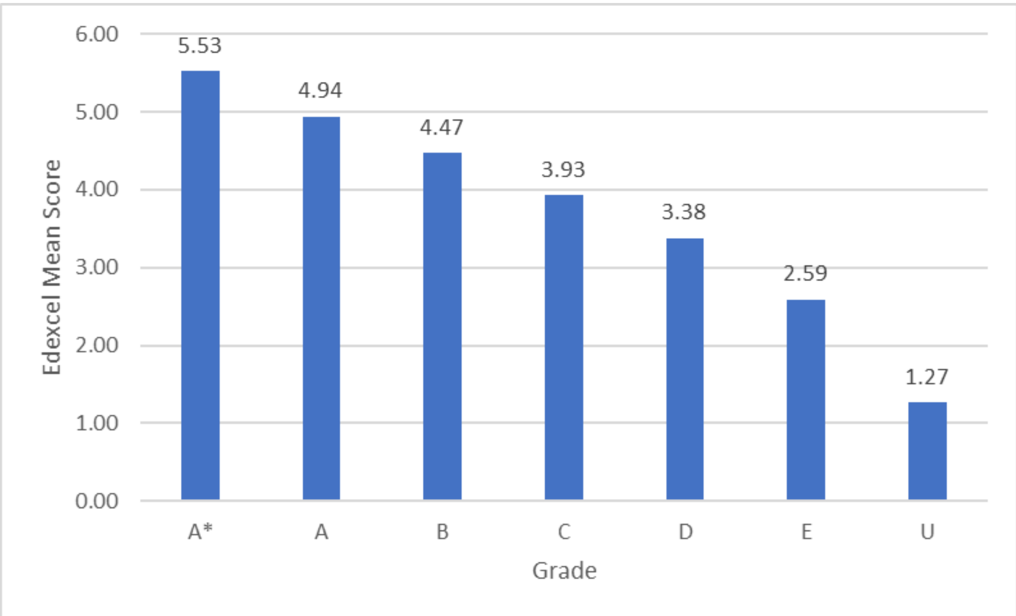
In part (b) (i) most candidates understood the order 4, periodic nature of the recurrence relationship and that $u_5=u_1=u_{4k-3}$, $k\in Z^+$. Occasionally, candidates unnecessarily calculated u_5, using u_4 in the recurrence relationship, which sometimes led to the mark being lost either if u_4 had an incorrect value or if an error was made in the calculation.

In part (ii) many candidates recognised how to construct the arithmetic calculation to find the required sum. The most common of these attempts were: $6(u_1+u_2+u_3+u_4)+u_1$ and $6(u_2+u_3+u_4)+7u_1$

Credit was given to candidates who used such ideas with incorrect values from earlier in the question for u_3 and u_4, although those who had a wrong answer in (a)(i) who used their incorrect value for u_2 (rather than the given $u_2=40$), lost both marks. It is important that centres make candidates aware that they must use the 'show that' answer in subsequent calculations no matter what they achieve in their working. Occasionally, candidates used $\sum_{r=1}^{25} u_r = 6.25(u_1+u_2+u_3+u_4)$ but these usually did not add 1 to give a fully correct method. Some also incorrectly tried summing the series as though the terms formed an arithmetic progression. Some candidates incorrectly multiplied ($u_1 + u_2 + u_3 + u_4$) by 4 instead of by 6. A minority forgot to add 35 to their total. A very small minority of candidates generated all the terms of the whole sequence and added them together.

Question 2 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
4.27	6	71%	5.53	4.94	4.47	3.93	3.38	2.59	1.27



Question 2 - Response A

$u_2 = 35 + 7\cos\left(\frac{35\pi}{2}\right) - 5(-1)^{35}$

$= 35 + 0 + 5$

$= 40$

$u_3 = \cancel{35}\ 40 + 7\cos\left(\frac{40\pi}{2}\right) - 5(-1)^{40}$

$= 42$

$u_4 = 42 + 7\cos\left(\frac{42\pi}{2}\right) - 5(-1)^{42}$

$= 30$

$u_5 = \cancel{40}\ 35$

$\sum_{r=1}^{25} 35$

1/6 marks

Part (a)(i)
B0: This candidate uses an incorrect method. They use the value of the first term for *n*.

Part (a)(ii)
M0: This candidate makes the same error as they did in part (i)
A0: Follows M0

Part (b)
B1: Identifies the value of the 5th term correctly.
M0: No method shown for the sum
A0: Follows M0

Question 2 - Response B

a) i) $u_2 = 35 + 7\cos\left(\frac{\pi}{2}\right) - 5(-1)^1$

$= 40$

ii) $u_3 = 40 + 7\cos\left(\frac{2\pi}{2}\right) - 5(-1)^2 = 28$

$u_4 = 28 + 7\cos\left(\frac{3\pi}{2}\right) - 5(-1)^3 = 33$

b) i) 35, 40, 28, 33, 38

$u_5 = 38$

ii)

3/6 marks

Part (a)(i)
B1: Correct work showing that $u_2 = 40$

Part (a)(ii)
M1: Correct method shown for the 3rd term.
A1: Correct values

Part (b)
B0: Incorrect value for the 5th term.
M0A0: Not attempted.

Q2
?
A
B
C

Question 2 - Response C

a i) $u_1 = 35$

$u_2 = 35 + 7\cos\left(\frac{0\pi}{2}\right) - 5(-1)^1 = 40$

ii) $u_3 = 40 + 7\cos\left(\frac{2\pi}{2}\right) - 5(-1)^2 = 28$

$u_4 = 28 + 7\cos\left(\frac{3\pi}{2}\right) - 5(-1)^3 = 33$

$u_3 = 28 \qquad u_4 = 33$

b i) $u_5 = 35$

ii) $u_1 + u_2 + u_3 + u_4 = 35 + 40 + 28 + 33 = 136$

$\cancel{(136 \times 4) + 35 = 579}$ $\qquad 24 + t$

$24 + 1 = 25$

$136 \times 6 + 35 = 851 \qquad \frac{24}{4} = 6$

6/6 marks

Part (a)(i)
B1: Correct work showing that $u_2 = 40$

Part (a)(ii)
M1: Correct method shown for the 3rd term.
A1: Correct values.

Part (b)
B1: Identifies the value of the 5th term correctly.
M1: A correct method shown for the sum.
A1: For 851

Q2
?
A
B
C

Question 3

Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 3 - Question

3. Given that

$$\log_2 (x + 3) + \log_2 (x + 10) = 2 + 2 \log_2 x$$

(*a*) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(*b*) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (*b*)(i) is not a solution of

$$\log_2 (x + 3) + \log_2 (x + 10) = 2 + 2 \log_2 x$$

giving a reason for your answer.

(2)

(Total for Question 3 is 5 marks)

Question 3 - Mark Scheme

Question	Scheme	Marks	AOs
3(a)	Uses one correct log law e.g. $\log_2(x+3)+\log_2(x+10)=\log_2(x+3)(x+10)$ $2=\log_2 4,\quad 2\log_2 x=\log_2 x^2$	M1	1.1b
	$(x+3)(x+10)=4x^2$ oe	dM1	2.1
	$\Rightarrow 3x^2-13x-30=0$ *	A1*	1.1b
		(3)	
(b)(i)	$(x=)\ 6,\ -\frac{5}{3}$	B1	1.1b
(ii)	$x\neq-\frac{5}{3}$ because $\log_{(2)}\left(-\frac{5}{3}\right)$ is not real	B1	2.4
		(2)	
		(5 marks)	

Question 3 - Examiner Comments

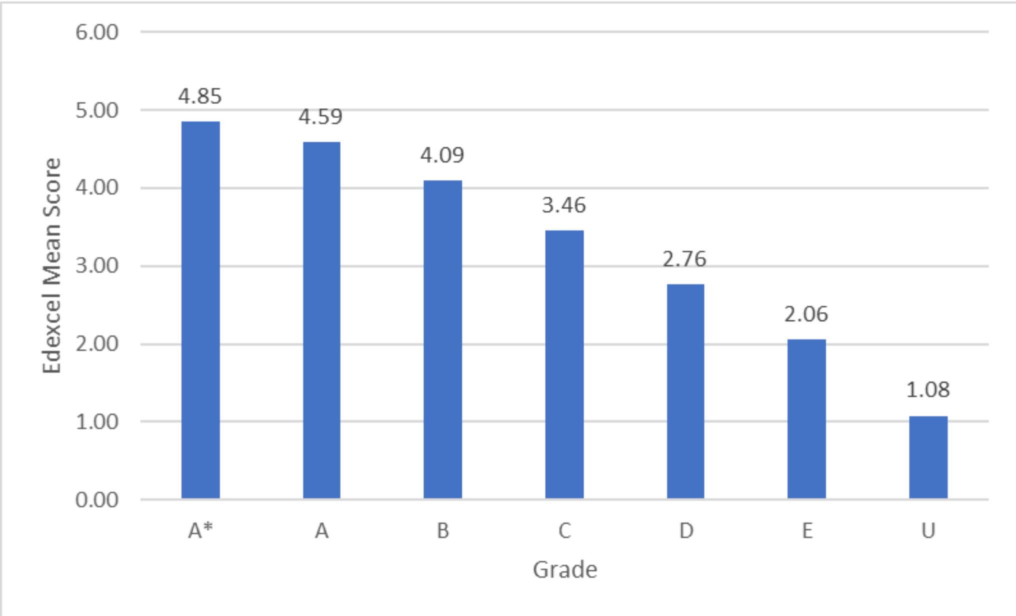
There were very few non-attempts or zero scores here and this question proved to be a good discriminator with a spread of marks achieved by candidates. Most students managed to score at least one mark out of three in part (a) by correctly applying a single log law, usually the product, division or power rules for logs. Dealing with the constant, 2, proved to be more challenging for many. Those who wrote 2 as $\log_2(4)$ and then applied the product rule for logs on the right-hand-side had most success. Others grouped the three log terms and applied the exponential rule to eliminate the logs. Sometimes there was a lot of working which wasn't always set out as clearly as it could have been. A significant proportion of candidates did succeed, but it was not uncommon to see correct use of a log law followed by incorrect log work in an attempt to fudge the printed result. Some poor responses involved an initial 'cancelling' of logs before any attempt to combine log terms or to raise both sides to the base 2 correctly. There was a minority of candidates who perhaps knew how to proceed but did not show sufficient working to gain full marks in a 'show that' question. Candidates should be reminded that all steps should be shown in this type of question. It was a shame when fully correct log work was followed by an incomplete 'equation' at the end of this part, as missing the '$= 0$' meant the loss of the final mark here. A small number of candidates who were unsure how to start part (a) used the printed answer and tried to "work backwards" but often made errors leading to incorrect solutions.

In (b) part (i), almost all candidates were able to correctly state the values '6' and '$-\frac{5}{3}$', it was clear that in many cases candidates were making use of their calculators to solve the quadratic equation which was acceptable. Part (ii) however, was answered less reliably. Although most candidates had some understanding of the domain of a log function and knew that $x = -\frac{5}{3}$ was not a valid solution, many were unable to provide reasoning that was precise enough for the mark here. It was common to see, for example, "a log can't be negative", which was not considered sufficiently clear. Others stated simply 'because it is negative' which was also insufficient. A range of responses which engaged specifically with the domain of a log function needing to be positive were considered acceptable such as; 'log of a negative value is undefined/impossible/gives a math error'.

Q3
?
A
B
C

Question 3 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade: A*	A	B	C	D	E	U
3.81	5	76%	4.85	4.59	4.09	3.46	2.76	2.06	1.08



Question 3 - Response A

(2)

a) $\log_2 (x+3)(x+10) = 2+2\log_2 x$

$\log_2 x(x+10)+3(x+10) = 2+2\log_2 x$

$\log_2 x^2+10x+3x+30$

b)

i) $3x^2-13x-30=0$

$x=6 \qquad x=-\frac{5}{3}$

ii) $x=-\frac{5}{3}$ is not a solution as x cannot be negative.

2/5 marks

Part (a)
M1: One correct log law is seen applied to the left-hand side of the equation.
dM0: Does not reach an equation not involving logarithms.
A0: Follows M0

Part (b)(i)
B1: Correct values.

Part (b)(ii)
B0: Identifies the correct value but does not relate the value to logarithms.

Q3 ? A B C

Question 3 - Response B

3)a) $\log_2(x+3) + \log_2(x+10) = 2 + \log_2 x^2$

$\log_2(x^2 + 33x + 30) = 2 + \log_2 x^2$

b)i) $(3x + 5)(x - 6)$

$x = -5/3$ or $x = 6$

ii) $x = -5/3$ as you can't do $\log_2(x)$ of a negative value for x.

3/5 marks

Part (a)
M1: One correct log law is seen applied to the right-hand side of the equation.
dM0: Does not reach an equation not involving logarithms.
A0: Follows M0

Part (b)(i)
B1: Correct values.

Part (b)(ii)
B1: Identifies the correct value and provides an acceptable explanation.

Q3

?

A

B

C

Question 3 - Response C

a) $\log_2((x+3)(x+10)) - \cancel{2}\log_2 x^2 = 2$

$\log_2\left(\frac{(x+3)(x+10)}{x^2}\right) = 2$

$\frac{(x+3)(x+10)}{x^2} = 2^2$

$x^2+13x+30 = 4x^2$

$3x^2-13x-30=0$

bi) $x=6$ or $x=-5/3$ $\quad (x-6)(3x+5)$

ii) Cant be $-5/3$ since

$= 2\log -5/3$ cant be negative inside of a logarithm.

5/5 marks

Part (a)
M1: At least one correct log law is seen applied to the equation.
dM1: Reaches an equation not involving logarithms from correct work that is not the printed answer.
A1: Correct proof.

Part (b)(i)
B1: Correct values.

Part (b)(ii)
B1: Identifies the correct value and gives an acceptable explanation.

Question 4

Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 4 - Question

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(*a*) State the value of A.

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(*b*) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

(3)

(Total for Question 4 is 4 marks)

Question 4 - Mark Scheme

Question	Scheme	Marks	AOs
4(a)	$(A=)\ 55$	B1	3.4
		(1)	
(b)	$\left\{\frac{dH}{dt}=\right\}-ABe^{-Bt}$ or $\left\{\frac{dH}{dt}=\right\}-\text{"55"}Be^{-Bt}$	M1	3.1b
	$-B\times\text{"55"}=-7.5\Rightarrow B=\ldots\ \left(\frac{3}{22}=\text{awrt } 0.136\right)$	M1	1.1b
	$H=55e^{-0.136t}+30$	A1cso	3.3
		(3)	
		(4 marks)	

Question 4 - Examiner Comments

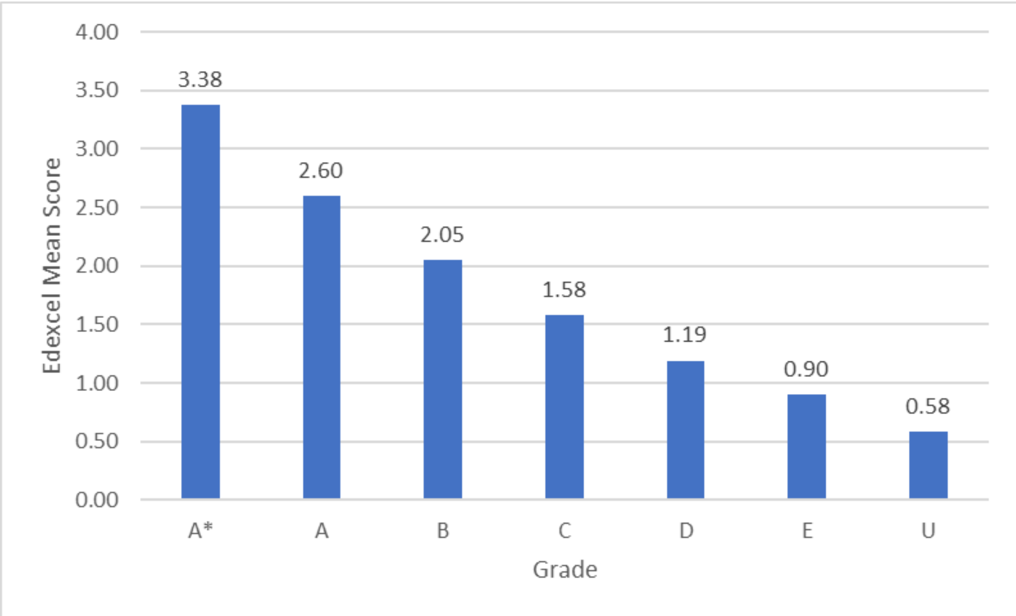
Those scoring full marks in this question were in a significant minority of candidates.

In part (a) the majority of candidates used the given information to find the value of A. Those that did not achieve this mark typically gave 85 as an answer. A few students failed to recognise the standard technique of substituting $t = 0$ into the expression for H to find the value of A, often using $t = 1$. Sometimes e^0 was evaluated as e or 0 rather than 1.

Part (b) had mixed outcomes for candidates. Interpretation of the given initial rate of cooling was problematic for many candidates. There was a general lack of appreciation that the *rate of cooling* was a continuous variable which represented $-\frac{dH}{dt}$. Those candidates not recognising this and using a difference method (similar to those introduced at GCSE) between $t = 0$ and $t = 1$ were restricted to the initial B mark in part (a). The most common attempt at a response for these candidates was setting $H(1) = 85 - 7.5$ and working from there. Of those who recognised the need to first find $\frac{dH}{dt}$ and then substitute $t = 0$, many incorrectly used $\frac{dH}{dt} = 7.5$ rather than the correct $\frac{dH}{dt} = -7.5$ to express a negative increase in temperature. This led to the loss of the last mark. Another error was to forget to differentiate the + 30 to zero leaving a constant in their $\frac{dH}{dt}$, only scoring at most the second method mark for substituting $t = 0$ and their A into their $\frac{dH}{dt}$. Some differentiated incorrectly to obtain $-55Bte^{-Bt}$. Several students missed out on the final mark despite correct work by not writing out the final equation. It is important that centres make candidates aware that when a question asks for the complete equation for a model that it is given as such and not just the values of any required constants.

Question 4 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
2.07	4	52%	3.38	2.60	2.05	1.58	1.19	0.90	0.58



Question 4 - Response A

a) $85 = Ae^{-B(0)} + 30$

$85 = A(1) + 30 \quad \Rightarrow \quad A = 55$

b) $H = 55e^{-Bt} + 30$

$\Rightarrow H = 55e^{-Bt(-7.5)} + 30$

$\Rightarrow$ ~~log~~ $\ln(H) = 55Bt(-7.5) + \ln(30)$

1/4 marks

Part (a)
B1: Correct value for *A*

Part (b)
M0: There is no attempt to differentiate.
M0: Does not satisfy the conditions stated in the mark scheme.
A0: Follows M0

Question 4 - Response B

(3)

a) $t = 0 \quad H = 85$

$85 = A + 30$

$A = 55°C$

b) $H = 55e^{-Bt} + 30$

$\frac{dH}{dt} = -55Be^{-Bt}$

$-55Be^{-Bt} = 7.5$

$Be^{-Bt} = \frac{-3}{22}$

2/4 marks

Part (a)
B1: Correct value for *A*.

Part (b)
M1: Differentiates to the required form.
M0: Does not substitute $t = 0$ and solve for B.
A0: Follows M0.

Q4

?

A
B
C

Question 4 - Response C

4a) when $t=0$ $H=85$

$85 = Ae^{-(0)B} + 30$

$85 = A + 30$

$A = 55$

b) so $\frac{dH}{dt} = 55(-B)e^{-Bt}$

when $t=0$, $\frac{dH}{dt} = -7.5$

so $-7.5 = -55Be^{-0}$

$-7.5 = -55B$

$B = \frac{7.5}{55}$

$B = \frac{3}{22}$

$B = 0.136$ (3dp)

so $H = 55e^{-0.136t} + 30$.

4/4 marks

Part (a)
B1: Correct value for *A*.

Part (b)
M1: Differentiates to the required form.

M1: Substitutes $t = 0$ and their A into $\frac{dH}{dt}$, sets $= \pm 7.5$ and solves for B.

A1: Correct equation written down.

Examiner comments
Note that the demand of the question in part (b) was to find a complete equation, not just the values of the constants.

Q4 ? A B C

Question 5

Question 5 - Question

5. The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point P (3, –10)
- has a turning point at P

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(*a*) show that $k = 12$

(2)

(*b*) Hence find the coordinates of the point where C crosses the y-axis.

(3)

(Total for Question 5 is 5 marks)

Question 5 - Mark Scheme

Question	Scheme	Marks	AOs
5(a)	$2(3)^3 - 9(3)^2 + 5(3) + k = 0 \Rightarrow k = ...$	M1	1.1b
	$54 - 81 + 15 + k = 0 \Rightarrow k = 12^*$ or $-12 + k = 0 \Rightarrow k = 12^*$	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3 - 9(3)^2 + 5(3) + 12 = 0$	M1	1.1b
	$54 - 81 + 15 + 12 = 0$ Hence $k = 12$ *	A1*	1.1b
		(2)	
(b)	$\int (2x^3 - 9x^2 + 5x + 12)\,dx ... x^4 \pm ... x^3 \pm ... x^2 \pm ... x \pm ...$	M1	3.1a
	$\frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c = -10 \Rightarrow c = ...$	dM1	1.1b
	$(0, -28)$	A1	2.2a
		(3)	
			(5 marks)

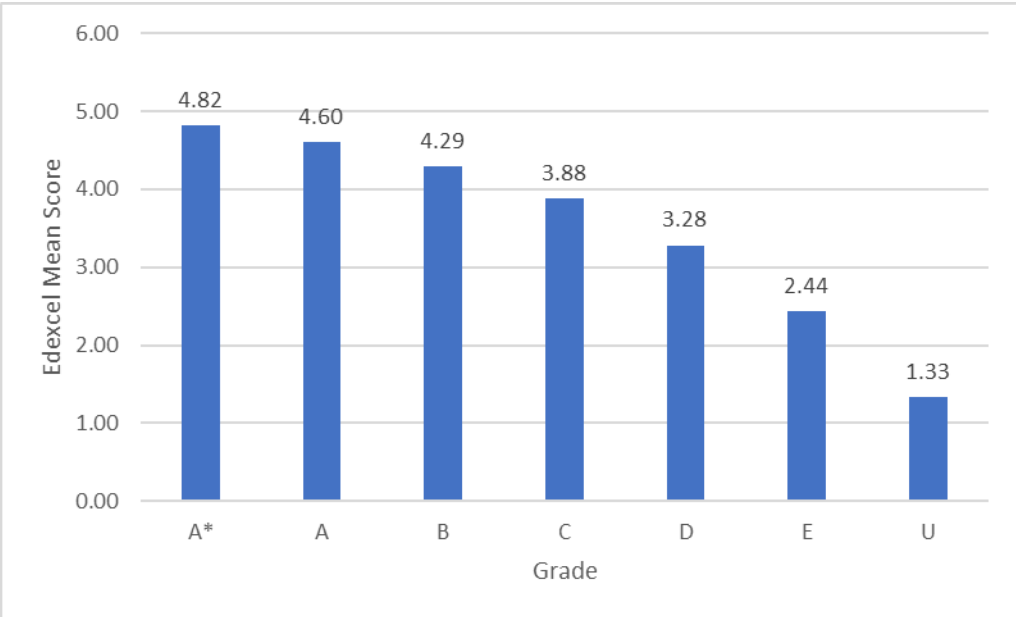
Question 5 - Examiner Comments

Part (a) of this question was accessible for the majority of candidates. Most understood that the derivative at a stationary point is zero and were able to achieve both the method mark and the accuracy mark, substituting $x = 3$ into the given derivative and setting it equal to zero. Occasionally the accuracy mark was lost due to insufficient working following substitution e.g. $2(3)^3 - 9(3)^2 + 5(3) + k = 0$ followed by $k = 12$ thus omitting a required intermediate step. In a 'show that' question candidates need to ensure that they have completely justified the given answer. Occasionally students showed the substitution of $x = 3$ into the derivative, processed the powers and simplified to obtain $k - 12$ and then jumped to $k = 12$. These responses lacked the appreciation that the correct answer had been achieved because the gradient equals 0 at $x = 3$. There were some rare instances of candidates incorrectly substituting $x = 3$ and $\frac{dy}{dx} = -10$ to find the value of k. Some candidates automatically integrated the given derivative but then realised that the differentiated expression was required in order to answer part (a) which they then successfully did. Other candidates integrated the expression and then substituted in the value for $k = 12$ to achieve an answer for part (b). Occasionally they then used the answer to part (b) to find $k = 12$, so were unable to access the marks available for this part of the question. A rarely seen alternative was to use algebraic division to divide the given polynomial by the linear factor $(x - 3)$ to achieve a remainder of $-12 + k$, and then equate that remainder to zero. This approach was met with varying degrees of success and students were often unable to complete the algebraic division successfully.

In part (b), candidates generally understood that they were required to integrate the expression and the straightforward integration meant the first method mark was commonly achieved. Most realised that they then needed to find the value of their constant of integration using $x = 3$ and $y = -10$ to determine the point where the curve crossed the y-axis and achieved the second method mark. The majority of students were successful in achieving full marks in this part, with most stating their answer in coordinate form $(0, -28)$ though some candidates left their answer as $c = -28$, not making the link with the y intercept explicit. Very few candidates made numerical slips in their processing to find the value of their constant of integration. Where mistakes were made, a common reason for not scoring the second method mark was due to substituting in a value of 0 instead of -10 for y in the integrated expression. Where the constant of integration had been omitted candidates went directly to substituting $x = 0$. Less commonly, some candidates differentiated rather than integrated and tried to find the roots of the second derivative rather than the y intercept.

Question 5 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
4.02	5	80%	4.82	4.60	4.29	3.88	3.28	2.44	1.33



Question 5 - Response A

Q5) $P(3, -10)$

$\int 2x^3 - 9x^2 + 5x + K$

$= \frac{1}{2}x^4 - 3x^2 + \frac{5}{2}x^2 + Kx + c.$

$P(3, -10)$

$-10 = \frac{1}{2}(3)^4 - 3(3)^2 + \frac{5}{2}(3)^2 + 3K + c.$

$+10 = 36 + 3K + c$

$+46 = 3K + c$

$+12 = K$

$\therefore 12 = K.$

b) $x = 0.$

So $\therefore$

1/5 marks

Part (a)
M0: No attempt to substitute $x = 3$, set = 0 and solve for k.
A0: Follows M0

Part (b)
M1: For the correct powers of at least 2 terms following integration.
M0: This candidate has 2 constants in their equation and so this mark is not available.
A0: Follows M0.

Examiner comments
Note parts (a) and (b) were marked together so that credit could be given to candidates who, for example, demonstrated integration skills in part (a).

Question 5 - Response B

(a) P – turning point $\frac{dy}{dx} = 0$

$0 = 2x^3 - 9x^2 + 5x + k \qquad x = 3$

$0 = 2(3)^3 - 9(3)^2 + 5(3) + k$

$0 = -12 + k$

$k = 12$

(b) crosses y-axis $x = 0$

$F(x) = \int 2x^3 - 9x^2 + 5x + 12$

$= \frac{2x^4}{4} - \frac{9x^3}{3} + \frac{5x^2}{2} + 12x$

$F(x) = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x$

$y = \frac{1}{2}(0)^4 - 3(0)^3 + \frac{5}{2}(0)^2 + 12(0)$

$y = 0 \qquad (0, 0)$

3/5 marks

Part (a)
M1: Substitutes $x = 3$, sets $= 0$ and solves for k.
A1: Obtains $k = 12$ with no errors. Note that the working was minimally acceptable.

Part (b)
M1: For the correct powers of at least 2 terms following integration.
M0: This candidate substitutes $x = 0$ which is an incorrect method.
A0: Follows M0.

Q5
?
A
B
C

Question 5 - Response C

$0 = 2(3)^3 - 9(3)^2 + 5\times 3 + k$

$0 = -12 + k$

$k = 12$

$\underbrace{\frac{2}{4}x^4 - 3x^3 + \frac{5}{2}x^2}_{-18} + 12x + c$

$-10 = -18 + 36$

$-10 = 18 - c$

$c = -28$

y intercept $= -28$

5/5 marks

Part (a)
M1: Substitutes $x = 3$, sets $= 0$ and solves for k.
A1: Obtains $k = 12$ with no errors. Note that the working was minimally acceptable.

Part (b)
M1: For the correct powers of at least 2 terms following integration.
M1: For substituting $x = 3$ into their integrated expression, sets $= -10$ and solves for their constant of integration.
A1: Correct value for y. (Note that this mark was not awarded for just $c = -28$)

Q5
A
B
C

Question 6

Question 6 - Question

6. Relative to a fixed origin O,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(*a*) Show that AD is parallel to BC.

(2)

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(*b*) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

(Total for Question 6 is 6 marks)

Question 6 - Mark Scheme

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AD} = 10\mathbf{i} + 24\mathbf{j}$ and $\overrightarrow{BC} = 50\mathbf{i} + 120\mathbf{j}$	M1	1.1b
	$\overrightarrow{AD} = \frac{1}{5}\overrightarrow{BC}$ therefore AD is parallel to BC *	A1*cso	2.2a
		(2)	
(b)	Attempt to find at least two lengths between AB, BC, CD and AD $\lvert\overrightarrow{BC}\rvert = \sqrt{50^2+120^2} = 130$, $\lvert\overrightarrow{DA}\rvert = \sqrt{10^2+24^2} = 26$	M1	1.1b
	$\lvert\overrightarrow{AB}\rvert = \sqrt{12^2+16^2} = 20$, $\lvert\overrightarrow{CD}\rvert = \sqrt{28^2+112^2} = 28\sqrt{17}$ (awrt 115 m)	A1	1.1b
	Average speed $= \dfrac{2\left(26+130+20+28\sqrt{17}\right)/1000}{5/60}$	dM1	3.1b
	awrt $= 6.99$ (km/h)	A1	3.2a
		(4)	
		(6 marks)	

Question 6 - Examiner Comments

This question was relatively well attempted. Most candidates attempted both parts and scored highly overall.

In part (a) the majority of candidates correctly identified $10\mathbf{i} + 24\mathbf{j}$, and $50\mathbf{i} + 120\mathbf{j}$. Those who didn't, often mistakenly added the position vectors instead of subtracting them. Most then went on to show that one was a multiple of the other (or that they were both different multiples of the same vector). However, some wrote the vectors the wrong way round for their multiple, which lost the second mark. Some candidates used other successful methods involving ratios or "gradients".
A common error here was candidates attempting to conclude the vectors were parallel by calculating and using their magnitudes. The most common reason for the loss of a mark in part (a) was with candidates correctly showing that the two vectors were multiples of each other but then not drawing a conclusion afterwards. It is important that candidates state what is shown by their working and provide a conclusion.

In part (b) most candidates correctly identified the lengths of *AB*, *BC*, *CD* and *AD*, but many left out their calculations by not showing the application of Pythagoras' Theorem. This is an important step that should be included to fully demonstrate where their answers came from. There was a misunderstanding by some students who thought they could add the 4 vectors of the sides together to calculate a total vector and work out the length of the resulting vector. A few instead found the lengths of the position vectors and added them. Some also assumed that there were two pairs of equal length lines and/or two pairs of parallel lines and just used the two lengths from part (a) then doubled them. Despite this, candidates often achieved the first two marks. The calculation of the average speed was also done well by many candidates. Although several candidates based their calculation on a single lap and arrived at half the correct answer, the majority completed it correctly and clearly showed division by a fraction equivalent to $\frac{5}{60}$ (most commonly $\frac{1}{12}$) or multiplication by its reciprocal. Candidates should be aware of the general instruction in the rubric to give non-exact answers to 3 significant figures; it was common to see 7 or 7.0 instead of the correct final answer.

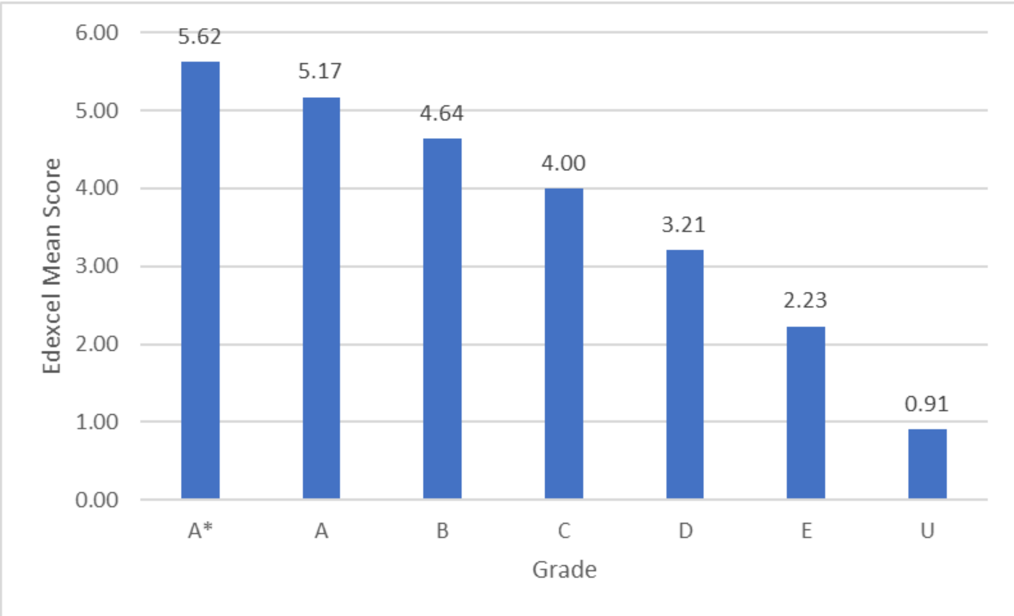
Q6

?

A
B
C

Question 6 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
4.32	6	72%	5.62	5.17	4.64	4.00	3.21	2.23	0.91



Question 6 - Response A

$\vec{AD} = 22i + 24j - 12i$

$\vec{AD} = 10i + 24j$

$\vec{BC} = 50i + 136j - 16j$

$BC = 50i + 120j$

$AD = 2(5i + 12j)$ equal terms

$BC = 10(5i + 12j)$

Q6
A
B
C

$|AB| = \sqrt{12^2 + 16^2} = 20$

$|BC| = \sqrt{50^2 + (136+16)^2} = 2\sqrt{6401}$

$|CD| = \sqrt{72^2 + 160^2} = 8\sqrt{481}$

$|DA| = \sqrt{34^2 + 24^2} = 2\sqrt{433}$

5 min for 2 laps

$\frac{60}{5} = 12$

1 hour for 24 laps

each lap $= 20 + 2\sqrt{6401} + 8\sqrt{481} + 2\sqrt{433}$ m

$= 397.0583501 2\ldots$

$\times 12 = 4765.002014$ m each hour

$= 4.765002014 \text{ kmh}^{-1}$

4.765 kmh^{-1}

1/6 marks

Part (a)
M1: Attempts to find vectors *AD* and *BC* using subtraction.
A0: There is no conclusion.

Part (b)
M0: This candidate uses vector addition not subtraction to find the lengths.
A0: Follows M0
dM0: This depends on the previous method mark.
A0: Follows M0.

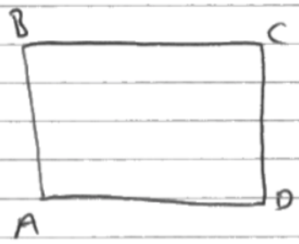
Question 6 - Response B

(4)

a) $\vec{AD} = \begin{pmatrix}22\\24\end{pmatrix} - \begin{pmatrix}12\\0\end{pmatrix} = 10i + 24j$

$\vec{BC} = \begin{pmatrix}50\\136\end{pmatrix} - \begin{pmatrix}16\\0\end{pmatrix} = 34i + 136j$

b)



$|AB| = \sqrt{12^2 + 16^2} = 20$

$|BC| = \sqrt{34^2 + 136^2} = 34\sqrt{17}$

$|CD| = \sqrt{(-28)^2 + (-112)^2} = 28\sqrt{17}$

$|DA| = \sqrt{10^2 + 24^2} = 26$

Total distance $= 46 + 62\sqrt{17}$ metres

$s = \frac{d}{t}$

$s = \frac{\left(\frac{46 + 62\sqrt{17}}{1000}\right) \times 2}{5} = 0.121 \text{ km s}^{-1}$

4/6 marks

Part (a)

M1: Attempts to find vectors *AD* and *BC* using subtraction. Note that this candidate has misread the position vector of *B* as 16**i** not 16**j**.

A0: Not available because of misreading the position vector of *B*.

Part (b)

M1: For finding at least 2 lengths of the quadrilateral

A1: The 2 lengths *AD* and *CD* are correct for this particular case of misreading the position vector of *B*

dM1: For attempting to find the average speed.

A0: Incorrect

Q6

?

A

B

C

Question 6 - Response C

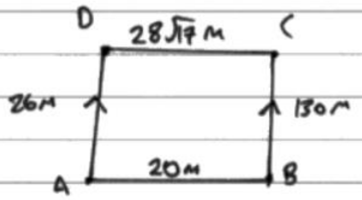
(4)

6a) $\vec{AD} = D - A = \begin{pmatrix}22\\24\end{pmatrix} - \begin{pmatrix}12\\0\end{pmatrix} = \begin{pmatrix}10\\24\end{pmatrix}$

$\vec{BC} = C - B = \begin{pmatrix}50\\136\end{pmatrix} - \begin{pmatrix}0\\16\end{pmatrix} = \begin{pmatrix}50\\120\end{pmatrix} = 5\begin{pmatrix}10\\24\end{pmatrix} = 5(\vec{AD})$

$\vec{AD}$ and $\vec{BC}$ share same direction vector so they're parallel.

b)



$2(AB + \overset{6AD}{\cancel{2BD}} + CD)$ takes 5 minutes

length of $AB = |B - A| = \left|\begin{pmatrix}0\\16\end{pmatrix} - \begin{pmatrix}12\\0\end{pmatrix}\right| = \left|\begin{pmatrix}-12\\16\end{pmatrix}\right| = \sqrt{12^2+16^2} = 20$

$|\vec{BC}| = \sqrt{50^2+120^2} = 130$ m

$|\vec{CD}| = \left|\begin{pmatrix}22\\24\end{pmatrix} - \begin{pmatrix}50\\136\end{pmatrix}\right| = \left|\begin{pmatrix}-28\\-112\end{pmatrix}\right| = 28\sqrt{17}$ m

$2(20 + 130 + 28\sqrt{17} + 26) = 582.894$ m

$|\vec{AD}| = \frac{1}{5}|\vec{BC}| = \frac{1}{5}(130) = 26$ m

time $= 300s = 5$ mins $= \frac{1}{12}$ hour

speed $= \frac{\text{distance}}{\text{time}} = \frac{582.894}{1/12} = \cancel{6994\text{ km/h}}$ $\cancel{7000\text{ km/h}}$ (2sf)

$\frac{582.893915}{1/12} = 6994.72698$ m/hour $= 6.99$ km/h (3sf)

6/6 marks

Part (a)
M1: Attempts to find vectors *AD* and *BC* using subtraction.
A1: Correct work and conclusion.

Part (b)
M1: For finding at least 2 lengths of the quadrilateral
A1: At least 2 lengths are correct.
dM1: For attempting to find the average speed.
A1: For 6.99 (note that units were not required but if they were given they must be correct)

Question 7

Question 7 - Question

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

(*b*) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(Total for Question 7 is 7 marks)

Question 7 - Mark Scheme

Question	Scheme	Marks	AOs
7(a)	$x^3 \to ...x^2$ and $3y^2 \to ...y\frac{dy}{dx}$	M1	1.1b
	$2xy \to 2y + 2x\frac{dy}{dx}$	B1	1.1b
	$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = ... \Rightarrow \frac{dy}{dx} = ...$	M1	2.1
	$\frac{dy}{dx} = -\frac{2y+3x^2}{2x+6y}$	A1	1.1b
		(4)	
(b)	$\frac{dy}{dx} = -\frac{2(5)+3(-2)^2}{2(-2)+6(5)}$ or e.g. $3(-2)^2 + 2(-2)\frac{dy}{dx} + 2\times5 + 6\times5\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = ...\ \left(-\frac{11}{13}\right)$	M1	1.1b
	$y - 5 = "\frac{13}{11}"(x+2)$	dM1	1.1b
	$13x - 11y + 81 = 0$	A1	2.2a
		(3)	
			(7 marks)

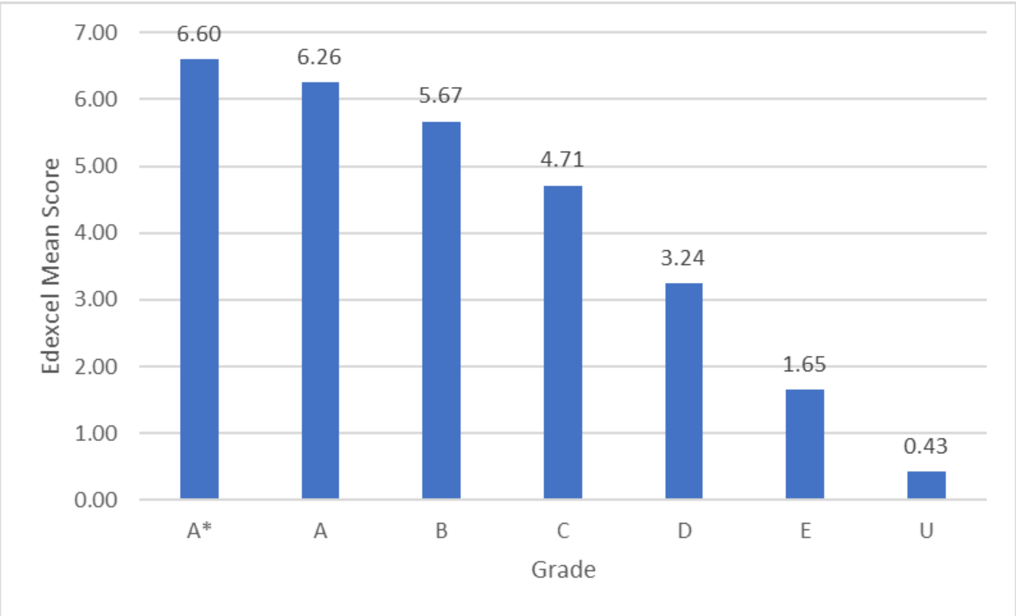
Question 7 - Examiner Comments

Almost all candidates attempted to use implicit differentiation with good levels of success. It was somewhat surprising that a significant number of candidates were let down by failing to differentiate the constant term correctly despite, in many cases, managing to correctly carry out some of the more complex steps in the implicit differentiation. It was not uncommon though to see errors in either the product rule for $2xy$ (often obtaining $\frac{d}{dx}(2xy) = 2\frac{dy}{dx}$ or $\frac{d}{dx}(2xy) = 2x\frac{dy}{dx}$ or errors in the application of the chain rule when differentiating $3y^2$. A surprising number of candidates set their first line of working equal to $\frac{dy}{dx}$ and fortunately this was often ignored in later working and so was condoned. Some candidates however, attempted to incorporate the extra $\frac{dy}{dx}$ term into their manipulations to find an expression for $\frac{dy}{dx}$ and this was costly. A few candidates struggled to deal with the negative terms in their numerator (or denominator) which led to sign errors in part (b). Others made slips in the rearrangement which lost the final mark. In some cases, there were issues with clarity of notation for the final statement of $\frac{dy}{dx}$ in terms of the positioning of the minus sign and candidates should be advised to take care when drawing the vinculum, particularly when the starting term of the numerator is negative to make sure the sign is in the correct place.

In part (b), the majority of candidates were able to use the correct approach to find the equation of the normal and candidates were often able to earn at least some of the marks here even if they had been unsuccessful in part (a). The least successful approach for calculation of the gradient of the normal tended to involve an attempt to rearrange the expression for $\frac{dy}{dx}$ into $-\frac{dx}{dy}$ prior to substitution and this seem to be more error-prone than the alternative of substituting in values as a first. Some candidates did not spot that point P on the curve had been given in the question and so didn't use this point in order to find the equation of the line. In some cases, candidates found the equation of the tangent rather than the normal whilst others made arithmetical slips and lost the accuracy mark. Occasionally candidates did not state their equation in the required form which was a shame when it followed correct work.

Question 7 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
5.00	7	71%	6.60	6.26	5.67	4.71	3.24	1.65	0.43



Question 7 - Response A

a) $\sigma \leq x^3 + 2xy + 3y^2 - 47$ $\quad u = 2x \quad v = y$

$u' = 2 \quad v' = 1$

$\frac{dy}{dx} = 3x^2 + 2y\frac{dy}{dx} + 2x + 6y\frac{dy}{dx} =$

$\cancel{-3x^2 + 2x = 2y\frac{dy}{dx}}$

$\frac{dy}{dx} = 3x^2 + 2x + 8y\frac{dy}{dx}$

$-8y\frac{dy}{dx} = 3x^2 + 2x$

$\frac{dy}{dx} = -\frac{3x^2 + 2x}{8y}$

b) normal $\Rightarrow \frac{8y}{3x^2 + 2x}$

8 (

1/7 marks

Part (a)

M1: Differentiates x^3 and $3y^2$ to the correct forms.

B0: The attempt at the product rule is incorrect.

M0: The attempt to make $\frac{dy}{dx}$ the subject requires 2 different $\frac{dy}{dx}$ terms. Note that this candidate has a spurious "$\frac{dy}{dx} =$" at the beginning which is never used.

A0: Follows M0

Part (b)

M0: There is no attempt to substitute $x = -2$ and $y = 5$

dM0A0: Follows M0

Question 7 - Response B

a) $3x^2 + 2y + \frac{dy}{dx}2x + \frac{dy}{dx}6y = 47$

$3x^2 + 2y - 47 = \frac{dy}{dx}(-2x - 6y)$

$\frac{dy}{dx} = \frac{3x^2 + 2y - 47}{-2x - 6y}$

b) $P(-2,5)$ normal at P

$\frac{dy}{dx} = \frac{3(-2)^2 + 2(5) - 47}{-2(-2) - 6(5)} = \frac{25}{26}$

↳ m of tangent $= \frac{25}{26}$

↳ m of normal $= -\frac{26}{25}$

$y = -\frac{26}{25}x + c$

$5 = -\frac{26}{25}(2) + c$

$\frac{117}{25} = c$

~~25~~y $\quad y = -\frac{26}{25}x + \frac{117}{25}$

$25y + 26x - 117 = 0$

$a = 26 \quad b = 25 \quad c = -117$

4/7 marks

Part (a)

M1: Differentiates x^3 and $3y^2$ to the correct forms.

B1: Correct attempt at the product rule.

M1: The attempt to make $\frac{dy}{dx}$ the subject satisfies the conditions in the mark scheme.

A0: Incorrect expression for $\frac{dy}{dx}$ as the "47" was not differentiated.

Part (b)

M1: Substitutes $x = -2$ and $y = 5$ into their $\frac{dy}{dx}$

dM0: This candidate incorrectly uses $x = 2$ and not $x = -2$ when finding the equation of the normal.

A0: Follows M0.

Q7 ? A B C

Question 7 - Response C

a) $x^3 + 2xy + 3y^2 = 47$

$\cancel{3x^2}$

$u = 2x \qquad v = y$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = \frac{dy}{dx} \qquad 2x\frac{dy}{dx} + 2y$

$\cancel{3x^2 + 2x\frac{dy}{dx}\,2y}$

$3x^2 + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 6y\frac{dy}{dx} = -3x^2 - 2y$

$\frac{dy}{dx}(2x+6y) = -3x^2 - 2y$

$\frac{dy}{dx} = \frac{-3x^2 - 2y}{2x + 6y}$

Q7 ? A B C

b) ~~grar~~ $\frac{dy}{dx} = \frac{-3(-2)^2 - 2(5)}{2(-2)+6(5)}$

$= -\frac{11}{13}$

perpendicular gradient

$= \frac{13}{11}$

$y - 5 = \frac{13}{11}(x+2)$

$11y - 55 = 13(x+2)$

$11y - 55 = 13x + 26$

$13x - 11y + 81 = 0$

7/7 marks

Part (a)

M1: Differentiates x^3 and $3y^2$ to the correct forms.

B1: Correct attempt at the product rule.

M1: The attempt to make $\frac{dy}{dx}$ the subject satisfies the conditions in the mark scheme.

A1: Correct expression for $\frac{dy}{dx}$

Part (b)

M1: Substitutes $x = -2$ and $y = 5$ into their $\frac{dy}{dx}$

dM1: Correct attempt at the equation of the normal.

A1: Correct equation in the required form.

Question 8

Question 8 - Question

8.

(*a*) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(*b*) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

(Total for Question 8 is 6 marks)

Question 8 - Mark Scheme

Question	Scheme	Marks	AOs
8(a)	$R = \sqrt{2^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$	B1	1.1b
	$2\cos\theta + 8\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $2 = R\cos\alpha \quad 8 = R\sin\alpha$ $\tan\alpha = \frac{8}{2} \Rightarrow \alpha = \ldots$	M1	1.1b
	$\alpha =$ awrt 1.326	A1	2.2a
		(3)	
(b)(i)	$4.5 \times "2\sqrt{17}"$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
		(6 marks)	

Question 8 - Examiner Comments

Part (a) of this question was generally answered very well with candidates regularly scoring full marks. Candidates seemed to be well practiced at converting to harmonic form and working in radians. The most successful candidates wrote out the full expansion of $R\cos(\theta-\alpha)$ rather than trying to take a short cut. The common issues included giving a decimal solution for R or rounding the value of α to a lower degree of accuracy than was required in the question. There were a small number of candidates who worked with $\tan\alpha = \frac{2}{8}$ and so lost the method mark and the accuracy mark.

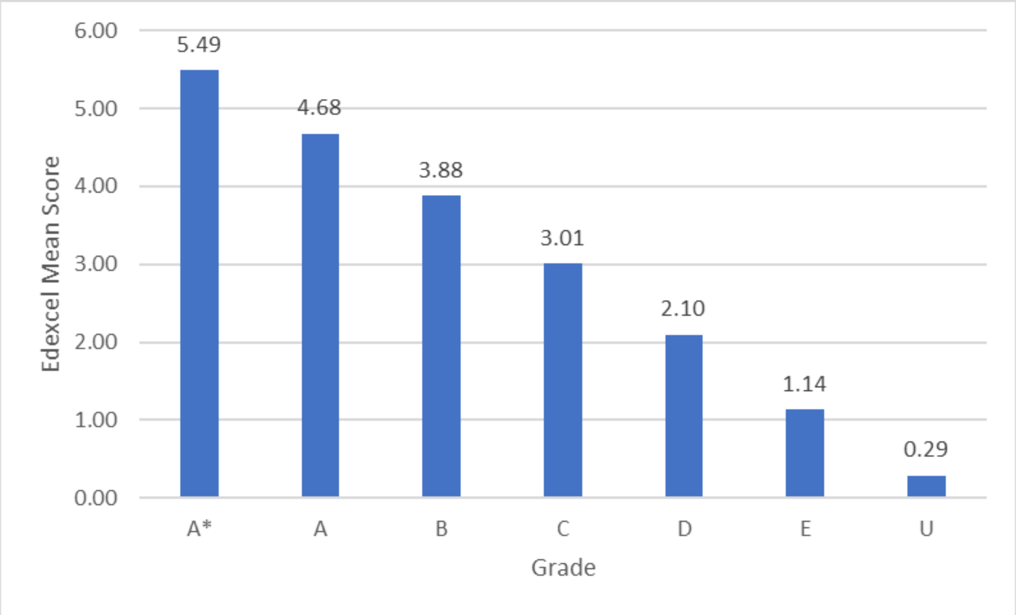
Candidates found part (b) more challenging, with many failing to link areas of mathematics, in this case trigonometry and series. Even where a candidate successfully found the sum,

$S = 9\cos x + 36\sin x$, many did not relate it to part (a). Those who did recognise the form were usually able to get $\pm 4.5\times "R"$, usually $\pm 4.5\times 2\sqrt{17}$ for part (i). Some seemed to misread the question and stated the minimum value, so lost the accuracy mark. A significant number of candidates found a correct expression in harmonic form but lost both the M1 and A1 as they kept a correct $9\sqrt{17}$ embedded in their expression.

Candidates were slightly more successful in part (ii). Most candidates who stated a correct value for the maximum, also got the value of x correct, more often than not giving it to the same accuracy as in part (a). A few benefitted from this being a follow through mark. Some candidates tried to find a subsequent value for α and were often out by π or 2π , or attempted to rearrange $x - 1.326 = \pi$. A few responses were seen where S was differentiated and set to zero to find a maximum. These candidates often lost the accuracy mark as they found an answer of 37.1. A lot of incorrect responses gave a maximum of 36 when $\sin x = 1$ or 45 from finding 36 + 9.

Question 8 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
3.63	6	61%	5.49	4.68	3.88	3.01	2.10	1.14	0.29



Question 8 - Response A

(8)(a) $R = \sqrt{2^2 + 8^2}$

$R = 2\sqrt{17}$

$\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{2}{8} \therefore \tan\alpha = \frac{1}{4}$

$\therefore \alpha = \tan^{-1}\left(\frac{1}{4}\right)$

$\alpha = 0.245$ (3.d.p)

$\therefore 2\sqrt{17}\cos(\theta - 0.245)$

(b)(i) $\dfrac{2\cos\theta + 8\sin\theta}{2} = \cos\theta + 4\sin\theta = \sqrt{17}\cos(\theta - 0.245)$

biggest $\cos\theta = 1$

Smallest $\cos\theta = -1 \Rightarrow S_n = 1+$

1/6 marks

Part (a)
B1: For the correct exact value of *R*.
M0: Incorrect attempt to find the value of α.
A0: Follows M0

Part (b)(i)
M0: Does not multiply their value of *R* by 4.5
A0: Follows M0.

Part (b)(ii)
B0: No value for *x*.

Question 8 - Response B

8a) $R = \sqrt{2^2+8^2} = 2\sqrt{17}$.

$R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 2$

$\cos\alpha = \frac{2}{2\sqrt{17}} = \frac{1}{\sqrt{17}}$

$\alpha = 1 \cdot 326$

$2\cos\theta + 8\sin\theta = 2\sqrt{17}\cos(\theta - 1 \cdot 326)$.

8bi) $a = \cos x$ $d = \sin x$

$S_9 = \frac{9}{2}(2\cos x + (9-1)\sin x)$

$S_9 = \frac{9}{2}(2\cos x + 8\sin x)$

$S_9 = 9\cos x + 36\sin x$. which is max when $\cos = 1$, $\sin = 0$.

$S_9 = 9(1) + 36(0)$

$S_9 = 9$. (max).

8bii) $\cos x = 1$ $\sin x = 0$.

$x = 0$, $x = 0$

so $x = 0$

3/6 marks

Part (a)
B1: For the correct exact value of *R*.
M1: Correct attempt to find the value of α.
A1: For $\alpha = 1.326$

Part (b)(i)
M0: Although this candidate finds S9 they do not use this correctly to find the maximum value.
A0: Follows M0.

Part (b)(ii)
B0: Incorrect value for *x*.

Examiner comments
Note that this candidate correctly uses cosine and their R to find α in part (a)

Q8 ? A B C

Question 8 - Response C

(3)

a) $R\cos(\theta-\alpha) = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$R\cos\alpha = 2 \qquad R\sin\alpha = 8$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{2}$

$\tan\alpha = \frac{8}{2}$

$\alpha = 1.326$ (3dp)

$R = \sqrt{2^2+8^2} = 2\sqrt{17}$

$2\sqrt{17}\cos(\theta - 1.326)$

b) i) $a = \cos x$

$d = \cos x + \sin x - \cos x = \sin x$

$n = 9$

$S = \frac{n}{2}(2a + (n-1)d)$

$= \frac{9}{2}(2\cos x + (9-1)\sin x)$

$= 4.5(2\cos x + 8\sin x)$

$= 4.5(2\sqrt{17}\cos(x - 1.326))$

$= 9\sqrt{17}\cos(x - 1.326)$

max $= 9\sqrt{17}$

ii) $\cos(x - 1.326) = 1$

$x - 1.326 = 0$

$x = 1.326$

$\cos(x - 1.326) = 9\sqrt{17}$

$x = 1.326$

6/6 marks

Part (a)
B1: For the correct exact value of *R*.
M1: Correct attempt to find the value of *α*.
A1: For $\alpha = 1.326$

Part (b)(i)
M1: Multiplies their value of *R* by 4.5
A1: Correct maximum value.

Part (b)(ii)
B1: Correct value for *x*.

Question 9

Question 9 - Question

9. The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6 \ln (t + 3) \qquad t > -3$$

(*a*) Show that a Cartesian equation for C is

$$y = A \ln (x + B) \qquad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y-axis at the point P

(*b*) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c \ln 5$$

where a, b and c are integers to be found.

(4)

(Total for Question 9 is 7 marks)

Question 9 - Mark Scheme

Question	Scheme	Marks	AOs
9(a) Way 1	$x=(t+3)^2-25$	M1	1.1b
	$\Rightarrow x+25=(t+3)^2 \Rightarrow (x+25)^{\frac{1}{2}}=(t+3) \Rightarrow y=...$	M1	2.1
	$y=6\ln(x+25)^{\frac{1}{2}} \Rightarrow y=3\ln(x+25)$	A1cso	1.1b
		(3)	
	(a) Way 2		
	$y=6\ln(t+3)=3\ln(t+3)^2$	M1	1.1b
	$y=3\ln(t+3)^2=3\ln(t^2+6t+9)=3\ln(x+16+9)$	M1	2.1
	$y=3\ln(x+25)$	A1cso	1.1b
	(a) Way 3		
	$y=6\ln(t+3)\Rightarrow\frac{y}{6}=\ln(t+3)\Rightarrow t+3=e^{\frac{y}{6}}\Rightarrow t=e^{\frac{y}{6}}-3$	M1	1.1b
	$x=\left(e^{\frac{y}{6}}-3\right)^2+6\left(e^{\frac{y}{6}}-3\right)-16\Rightarrow y=...$ or $x=\left(e^{\frac{y}{6}}-3+8\right)\left(e^{\frac{y}{6}}-3-2\right)\Rightarrow y=...$	M1	2.1
	$y=3\ln(x+25)$	A1cso	1.1b
	(a) Way 4		
	$x=(t+3)^2-25$	M1	1.1b
	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6}\Rightarrow\frac{3}{(t+3)^2}=\frac{3}{x+25}\Rightarrow y=3\ln(x+25)(+c)$	M1	2.1
	e.g. $t=0\Rightarrow x=-16, y=6\ln3\Rightarrow 6\ln3=3\ln(9)\Rightarrow c=0$ $y=3\ln(x+25)$	A1cso	1.1b
(b)	$x=0,\ y=3\ln25$ oe e.g. $6\ln5$	B1ft	2.2a
	$\frac{dy}{dx}=\frac{3}{x+"25"}\Rightarrow\frac{dy}{dx}=\frac{3}{0+"25"}\ \left(=\frac{3}{25}\right)$ **or** $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\left(\frac{6}{t+3}\right)}{2t+6}\Rightarrow\frac{\frac{6}{2+3}}{2\times2+6}\ \left(=\frac{6}{50}=\frac{3}{25}\right)$	M1	2.1
	$y-"3\ln25"="\frac{3}{25}"(x\{-0\})$	dM1	3.1a
	$25y-3x=150\ln5$	A1	2.2a
		(4)	
		(7 marks)	

Q9 ? A B C

Question 9 - Examiner Comments

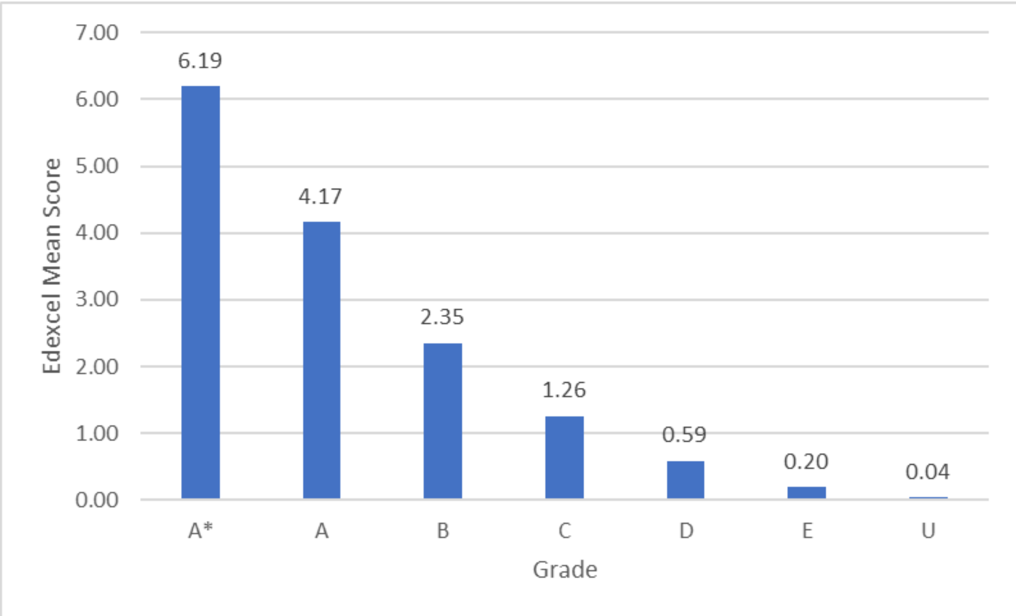
Most candidates found part (a) challenging but it was usually attempted. There were not many blank responses. Of the four methods listed on the mark scheme, one common successful method was to substitute t in terms of x into $y = 6\ln(t + 3)$ via "completing the square" (way 1). When substituting into $y = 6\ln(t + 3)$, if a correct t equation was created, then this usually led to full marks. Another common approach was to rearrange t in terms of y, usually successfully as $t = e^{\frac{y}{6}} - 3$ and substitute into $x = t^2 + 6t - 16$ (way 3). When using this method some candidates had difficulty squaring $e^{\frac{y}{6}}$ meaning they were unable to earn full marks. Some candidates incorrectly set $\sqrt{x+25} = \sqrt{x} + 5$ thus also losing the accuracy mark. A significant number of candidates factorised $x = t^2 + 6t - 16$ to give $x = (t - 2)(t + 8)$ and then wrongly thought $x = t - 2$. This then often led to $y = 6\ln(x + 5)$ which scored no marks in this part.

Candidates were generally more successful in part (b) than part (a). Many candidates found the y intercept by substituting $t = 2$ into the parametric equation for y rather than substituting $x = 0$ into the Cartesian equation. The first method mark was more challenging. Some candidates attempted the derivative from the Cartesian equation and they usually reached either $\dfrac{3}{(x+A)}$ or $\dfrac{3}{x}$ whilst others attempted to use the chain rule approach. Some candidates obtained the correct derivative but failed to substitute $x = 0$. If a candidate achieved the first method mark, they usually went on to achieve the dependent method mark for attempting to find the equation of the tangent. There were very few cases where the candidate used the negative reciprocal of their gradient. The most successful way candidates achieved full marks was differentiating their Cartesian equation. Those that differentiated parametrically were often caught out by arithmetic and algebraic errors or substituted $t = 0$ instead of $t = 2$ and failed to achieve the gradient correctly. For the final answer, some candidates had failed to realise the form of equation the question was asking for so left their answer as "$y =$" or they didn't have a, b and c as integers. Many candidates who had incorrectly answered (a), were able to go on to achieve full marks in (b) by using parametric differentiation to work out the gradient. There were very few blank responses for this part of the question.

Q9 ? A B C

Question 9 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
2.77	7	40%	6.19	4.17	2.35	1.26	0.59	0.20	0.04



Question 9 - Response A

⇒ $x+16 = (t+3)^2 - 9$

⇒ $x+25 = (t+3)^2$

⇒ $\sqrt{x+25} = t+3$ so $t = \sqrt{x+25} - 3$

⇒ at $t = \sqrt{x+25} - 3$; $y = 6\ln(\sqrt{x+25} - 3 + 3)$

$y = 6\ln(\sqrt{x} + 5)$

$A = 6$, $B = 5$

b) $\frac{dx}{dt} = 2t + 6$, $\frac{dy}{dt} = 6\ln\left(\frac{t+3}{1}\right)$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{6\ln(t+3)}{2t+6}$

2/7 marks

Q9 ? A B C

Part (a)
Following 'Way 1' from the mark scheme
M1: For attempting to complete the square on $t^2 + 6t - 16$
M1: Substitutes into the y coordinate correctly
A0: Incorrect. (This candidate does not deal with the square root correctly)

Part (b)
No valid work

Question 9 - Response B

9a. $x + 16 = t(t+6)$

$x + 16 = t^2 + 6t$

$\cancel{t(t+6)}$

$(t+3)^2 - \cancel{6}\,9$

$x + 16 = (t+3)^2 - 9$

$x + 25 = (t+3)^2$

$\sqrt{x+25} = t + 3$

$\sqrt{x+25} - 3 = t$

Question 9 continued

$\frac{dy}{dx}$ $\ln x$ $= \frac{1}{x}$

$\sqrt{x+25} - 3 = t$

$y = 6\ln(\sqrt{x+25} - 3 + 3)$

$y = 6\ln(\cancel{5}\sqrt{x} + 5)$

Q9

?

A
B
C

b. $\frac{dx}{dt} = 2t + 6$ $\qquad \frac{1}{t+3}$

$\frac{dy}{dt} = \cancel{6\ln(t+} \quad \frac{6}{t+3}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{6}{t+3} \times \frac{1}{2t+6}$

$\cancel{\frac{12+36t}{t+3}}$ $\cancel{\frac{dy}{dx}}$ $\frac{dy}{dx} = \frac{6}{(t+3)(2t+6)}$

$\cancel{12(1+3t)}$

$t = 2 \qquad t = -8$

$t = 2$

$y - y_1 = m(x - x_1)$

$y - 6\ln 5 = \frac{3}{25}(x + 16)$

$\cancel{y - 6}$

$25y - 150\ln 5 = 3x + 48$

$\cancel{25y - 3x}$

$3x - 25y = -150\ln 5 - 48$

4/7 marks

Part (a)

Following 'Way 1' from the mark scheme

M1: For attempting to complete the square on $t^2 + 6t - 16$ (Note they set $x + 16 = t^2 + 6t$ and then attempt this on the right-hand side)

M1: Substitutes into the y coordinate correctly.

A0: Incorrect. (This candidate does not deal with the square root correctly)

Part (b)

B1ft: $y = 6\ln 5$ is seen when they form the equation of the tangent.

M1: They use parametric differentiation correctly and substitute $t = 2$ (implied by their $\frac{3}{25}$)

dM0: The attempt at the equation is incorrect. They have $x + 16$ on the right-hand side not e.g. $(x - 0)$

A0: Follows M0

Question 9 - Response C

a) $x = t^2 + 6t - 16$ $\quad$ $x = (t+3)^2 - 9 - 16$

$(t+8)(t+2)$ $\quad$ $= (t+3)^2 - 25$

$x + 25 = (t+3)^2$

$\sqrt{x+25} = t+3$

$\sqrt{x+25} - 3 = t$

$y = 6\ln(\sqrt{x+25})$

$= 6\ln((x+25)^{1/2})$

$= 3\ln(x+25)$ $\quad$ $A = 3$ $\quad$ $B = 25$

b) $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$\dfrac{dx}{dt} = 2t + 6$ $\quad$ $\dfrac{dy}{dt} = \dfrac{6}{t+3}$

$\dfrac{dy}{dx} = \dfrac{6}{t+3} \times \dfrac{1}{2t+6} = \dfrac{6}{(t+3)(2t+6)}$

$3t^2 + 12t + 18$

$\dfrac{6}{3t^2+12t+18}$

$\dfrac{6}{3(2)^2+12(2)+18} = 1/9$

$y = 1/9\,x + 6\ln 5$

$9y = x + 54\ln 5$

$-x + 9y = 54\ln 5$

$a = -1$ $\quad$ $b = 9$ $\quad$ $c = 54$

6/7 marks

Part (a)
B1: For attempting to complete the square on $t^2 + 6t - 16$
M1: Substitutes into the y coordinate correctly.
A1: Correct equation with "$y =$" seen.

Part (b)
B1ft: $y = 6\ln 5$ is seen
M1: They use parametric differentiation correctly and substitute $t = 2$ seen where they obtain $\frac{1}{9}$
(note that they expand $(t + 3)(2t + 6)$ incorrectly but the intention is clear)
dM1: The attempt at the equation is correct.
A0: Incorrect because of the earlier slip when expanding brackets.

Examiner comments
This candidate makes a very good attempt at this question with the only slip being when expanding brackets in part (b).

Q9
?
A
B
C

Question 10

Question | Mark Scheme | Examiner Comments | Performance | Response A | Response B | Response C

Question 10 - Question

10. $$\mathrm{f}(x) = \frac{3kx - 18}{(x+4)(x-2)}$$ where k is a positive constant

(*a*) Express f (x) in partial fractions in terms of k.

(3)

(*b*) Hence find the exact value of k for which

$$\int_{-3}^{1} \mathrm{f}(x)\,\mathrm{d}x = 21$$

(4)

(Total for Question 10 is 7 marks)

Question 10 - Mark Scheme

Question	Scheme	Marks	AOs
10(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)}\equiv\frac{A}{x+4}+\frac{B}{x-2}\Rightarrow 3kx-18\equiv A(x-2)+B(x+4)$ **or** $\frac{3kx-18}{(x+4)(x-2)}\equiv\frac{A}{x-2}+\frac{B}{x+4}\Rightarrow 3kx-18\equiv A(x+4)+B(x-2)$	B1	1.1b
	$6k-18=6B\Rightarrow B=...$ or $-12k-18=-6A\Rightarrow A=...$ **or** $3kx-18\equiv(A+B)x+4B-2A\Rightarrow A+B=3k,\ -18=4B-2A$ $\Rightarrow A=...$ or $B=...$	M1	1.1b
	$\frac{2k+3}{x+4}+\frac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\int\left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right)dx=...\ln(x+4)+...\ln(x-2)$	M1	1.2
	$("2k+3")\ln(x+4)+("k-3")\ln(x-2)$	A1ft	1.1b
	$("2k+3")\ln(5)-("k-3")\ln(5)\Rightarrow("k+6")\ln 5=21\Rightarrow k=...$	dM1	3.1a
	$(k=)\frac{21}{\ln 5}-6$	A1	2.2a
		(4)	
		(7 marks)	

Q10

?

A
B
C

Question 10 - Examiner Comments

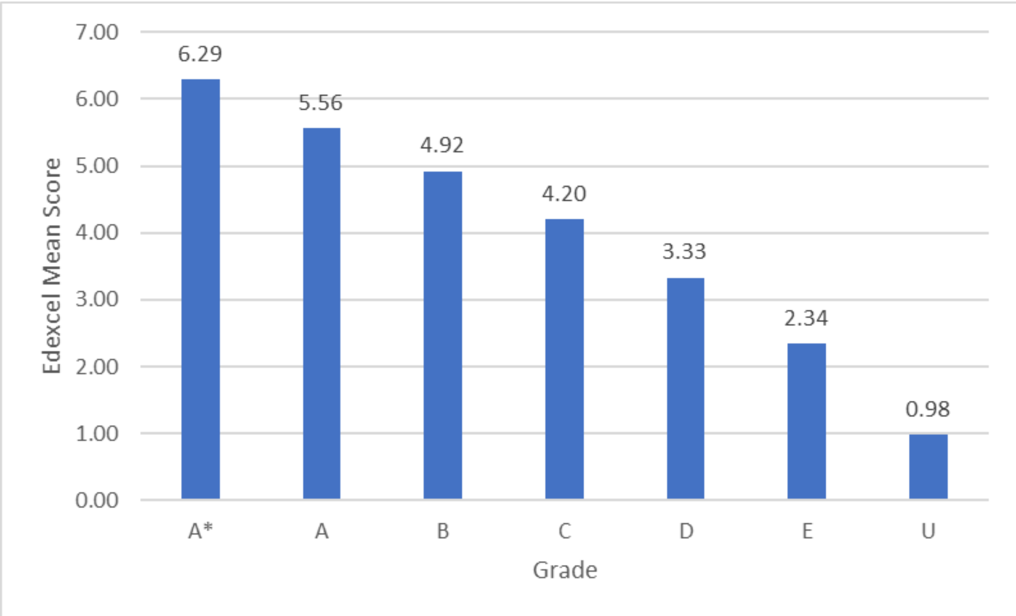
Part (a) was generally done well with most candidates not deterred by the occurrence of the "k" in the numerator. The majority of students correctly formed the partial fractions and set up the identity in terms of A and B. Once there, most of them correctly went forward by substituting x as 2 and – 4 and finding A and B in terms of k. Some set up their partial fractions in the form $A+\frac{B}{x+4}+\frac{C}{x-2}$ but very rarely managed to achieve $A = 0$. The method of comparing coefficients and solving two equations in A and B simultaneously was seen very occasionally. Errors were made where candidates had assigned the wrong values for A and B in their final partial fractions. A common sign error was made when substituting in $x = -4$, leading to solving $-12k - 18 = -6A$ to obtain $A = 2k - 3$, instead of $A = 2k + 3$. Some did not fully simplify their expressions for A or B, leaving them as fractions. Another error at this stage generally involved not substituting for x in the $3kx$ term, leaving A and B in terms of x. Some students transposed the $x - 2$ and $x + 4$ at the beginning so lost the B mark but generally followed through correctly and scored the method mark. In a small number of cases, the last mark was lost after candidates correctly found the numerators in terms of k, but did not write down the correct partial fractions.

Very few candidates managed full marks in part (b), although the majority did gain the first method mark for integrating to obtain $\ldots\ln(x + 4)$ or $\ldots\ln(x - 2)$. Those who did not recognise the logarithmic form for the integration were unable to score any marks in this part. Some achieved the correct form after first using a formal substitution. Of those who integrated their partial fractions successfully, the vast majority were far from strict in their use of the modulus symbol and many lost marks due to lack of appreciation of its importance. Use of modulus notation for integrating reciprocal functions should be picked up by centres as a teaching point with future cohorts. Students who had written the incorrect term $-(k-3)\ln(-5)$ or even the correct term $-(k-3)\ln|-5|$ after applying the limit $x=-3$, went on to indicate that they believed this term was zero or could just be ignored. Another example of poor practice was missing brackets around the coefficients e.g. $2k+3\ln(x+4)+k-3\ln(x-2)$ being surprisingly common. The "invisible" brackets were sometimes recovered but in many cases were not. There were also examples where the brackets around the $(x + 4)$ and the $(x - 2)$ were also missing. Those with a systematic well-laid out approach for the substitution of limits in both terms and identifying clear subtraction of the lower limit, went on to score 3 or 4 marks in part (b) and commonly full marks for the question. Many candidates however were hampered by their own layout making the correct interpretation of ln (–5) as ln |5| even more challenging. The perhaps unfamiliar, exact form needed for k also proved a challenge for even the best candidates. There were some elaborate attempts to achieve an expression for k, some using exponentials, but many gave up along the way. There were, however some very elegant and correct solutions to this question and some different forms of the exact equivalent answer were seen, for example $21\log_5 e-6$ but most correct answers were given as $\frac{21}{\ln 5}-6$.

Q10 A B C

Question 10 - Performance

Mean score	Max score	Mean %	Edexcel averages: mean scored by candidates achieving grade:						
			A*	A	B	C	D	E	U
4.64	7	66%	6.29	5.56	4.92	4.20	3.33	2.34	0.98



Q10
?
A
B
C

Question 10 - Response A

$$f(x) = \frac{3kx-18}{(x+4)(x-2)} =$$

$$\cancel{3kx - 18 = \frac{A}{x+4} + \frac{B}{x-2}}$$

$$\frac{3kx-18}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$$

$$3kx - 18 = A(x-2) + B(x+4)$$

$$\cancel{3kx - 18 = Ax - 2A + Bx + 4B}$$

$x = 2$

$$\cancel{3k} \quad 3k(2) - 18 = A(2-2) + B(2+4)$$
$$6k - 18 = 0 + 6B$$
$$6k - 18 = 6B$$

$x = -4$

$$3k(-4) - 18 = A(-4-2) + B(-4+4)$$
$$-12k - 18 = -6A$$

$$6k - 18 = 6B$$
$$\cancel{6k} \quad 6(-6) - 18 = 6B$$
$$-36 - 18 = 6B$$
$$-54 = 6B$$
$$B = -9$$

$$-12k - 18 = -6A$$
$$-12(-6) - 18 = -6A$$
$$54 = -6A$$
$$A = -9$$

1/7 marks

Part (a)

B1: Correct form for the partial fractions and sets up the correct corresponding identity.

M0: Incorrect method. This candidate uses substitution but then substitutes values of *k* to find *A* and *B*.

A0: Follows M0

Part (b)

Not attempted.

Q10

?

A

B

C

Question 10 - Response B

a) $\dfrac{3kx - 18}{(x+4)(x-2)} = \dfrac{A}{(x+4)} + \dfrac{B}{(x-2)}$

$3kx - 18 = A(x-2) + B(x+4)$

let $x = 2$

$3k(2) - 18 = B(2+4)$

$6k - 18 = 6B$

$k - 3 = B$

let $x = -4$

$\boxed{\dfrac{3kx - 18}{(x+4)(x-2)} = \dfrac{2k+9}{x+4} + \dfrac{k-3}{x-2}}$

$3k(-4) - 18 = A(-4-2)$

$-12k - 18 = -6A$

$A = 2k + 9$

b) $\displaystyle\int_{-3}^{1} \frac{2k+9}{x+4} + \frac{k-3}{x-2} = 21$

$= \left[2k+9 \ln|x+4| + k-3 \ln|x-2|\right]_{-3}^{1} = 21$

$2k + 9 \ln|1+4| + k - 3 \ln|1-2|$

$= 3k + 14.48$ ~~trw~~

$2k + 9 \ln|-3+4| + k - 3 \ln|-3-2|$

$2k + 0 + k - 4.83 = 3k - 4.83$

$3k + 14.48 + 3k - 4.83 = 6k + 9.65$

$6k + 9.65 = 21$

$6k = 11.35$

$k = \dfrac{227}{120}$

3/7 marks

Part (a)
B1: Correct form for the partial fractions and sets up the correct corresponding identity.
M1: Uses substitution to find at least one of their constants.
A0: The first fraction is incorrect.

Part (b)
M1: Attempts to integrate and achieves at least one expression of the required form. Note that missing brackets were condoned here.
A0ft: Incorrect follow through integration as the missing brackets are never recovered.
dM0: The limits are substituted but then the expressions are added not subtracted.
A0: Follows M0.

Examiner comments
Note that in part (a), errors when finding the numerator of the fraction with denominator $x + 4$ were seen quite frequently.

Question 10 - Response C

(4)

a) $A(x \mp 2) + B(x+4) = 3kx - 18$

let $x = 2$ $\quad B(6) = 6k - 18$

$B = k - 3$

let $x = -4$ $\quad A(-6) = -12k - 18$

$A = 2k + 3$

$$F(x) = \frac{2k+3}{x+4} + \frac{k-3}{x-2}$$

b) $$\int_{-3}^{1} \frac{2k+3}{x+4} + \frac{k-3}{x-2} \, . \, dx$$

$$\left[(2k+3\ln|x+4|) + (k-3\ln|x-2|) \right]_{-3}^{1}$$

$$\left[(2k+3)\ln|5| + (k-3)\ln|-1| \right] - \left[(2k+3)\ln 1 + (k-3)\ln|-5| \right]$$

$$\left[(2k+3)\ln 5 \right] - \left[(k-3)\ln 5 \right] = 21$$

$$\ln 5^{2k+3} - \ln 5^{k-3}$$

$$(2k\ln 5 + 3\ln 5) - (k\ln 5 - 3\ln 5) = 21$$

$$2k\ln 5 - k\ln 5 + 3\ln 25 = 21$$

$$k(2\ln 5) = 21 - 3\ln 25$$

$$k = \frac{21 - 3\ln 25}{\ln 5}$$

7/7 marks

Part (a)
B1: Correct form for the partial fractions and sets up the correct corresponding identity.
M1: Uses substitution to find at least one of their constants.
A1: Correct partial fractions.

Part (b)
M1: Attempts to integrate and achieves at least one expression of the required form.
A1: Correct integration
dM1: A full attempt to find the value of *k*.
A1: Correct value for *k*.

Q10 A B C

Question 11

Question 11 - Question

11.

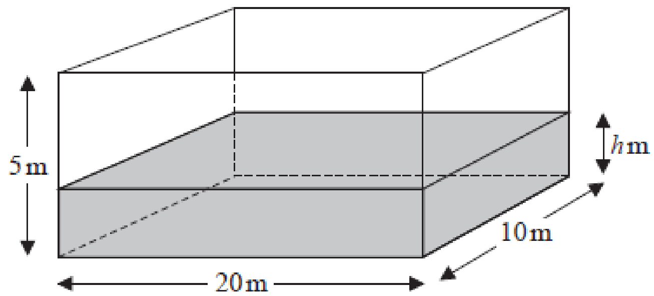


Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20 m by 10 m and the height of the tank is 5 m, as shown in Figure 1.

At time t minutes after water started flowing into the tank the height of the water was h m and the volume of water in the tank was V m^3

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(*a*) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3)

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(*b*) use the model to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5)

(*c*) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

(Total for Question 11 is 10 marks)

Question 11 - Mark Scheme

Question	Scheme	Marks	AOs
11(a)	$\frac{dV}{dh}=200$ oe e.g. $\frac{dh}{dV}=\frac{1}{200}$	B1	1.1b
	$\left(\frac{dh}{dt}=\right)\frac{dh}{dV}\times\frac{dV}{dt}=\frac{1}{200}\times\frac{k}{\sqrt{h}}$	M1	3.1a
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}$ *	A1*	2.1
		(3)	
(b)	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\int h^{\frac{1}{2}}\,dh=\int\lambda\,dt\Rightarrow ...h^{\frac{3}{2}}=\lambda t\{+c\}$	M1	1.1b
	$\frac{2}{3}h^{\frac{3}{2}}=\lambda t\{+c\}$ oe e.g. $\frac{h^{\frac{3}{2}}}{\frac{3}{2}}=\lambda t\{+c\}$	A1	1.1b
	$\frac{2}{3}(1.44)^{\frac{3}{2}}=\lambda\times0+c\Rightarrow c=1.152\left(=\frac{144}{125}\right)$	dM1	3.4
	$\frac{2}{3}(3.24)^{\frac{3}{2}}=\lambda\times8+\text{"}1.152\text{"}\Rightarrow\lambda=0.342\left(=\frac{171}{500}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
	(b) Alternative:		
	$\frac{dh}{dt}=\frac{\lambda}{\sqrt{h}}\Rightarrow\frac{dt}{dh}=\frac{\sqrt{h}}{\lambda}\Rightarrow t=...h^{\frac{3}{2}}(+c)$	M1	1.1b
	$t=\frac{2h^{\frac{3}{2}}}{3\lambda}(+c)$ oe	A1	1.1b
	$0=\frac{2(1.44)^{\frac{3}{2}}}{3\lambda}+c$ **and** $8=\frac{2(3.24)^{\frac{3}{2}}}{3\lambda}+c$ $\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **or** $c=...\left(-\frac{64}{19}\right)$	dM1	3.4
	$\Rightarrow\lambda=...\left(\frac{171}{500}\right)$ **and** $c=...\left(-\frac{64}{19}\right)$	ddM1	3.1b
	$h^{\frac{3}{2}}=0.513t+1.728$ oe e.g. $h^{\frac{3}{2}}=\frac{513}{1000}t+\frac{216}{125}$	A1	3.3
		(5)	
(c)	$5^{\frac{3}{2}}=0.513t+1.728\Rightarrow t=...$	M1	3.4
	$(t=)$ awrt 18.4 min	A1	3.2a
		(2)	
		(10 marks)	

Question 11 - Examiner Comments

Many candidates found this question challenging and there were a significant number of blank responses.

Candidates found part (a) particularly challenging, with many attaining either no marks or the first mark only. The concept of connected rates of change is one that candidates frequently struggle with. A good number found an expression for V and successfully differentiated to get $\frac{dV}{dh} = 200$, although some confused the volume of water with the volume of the tank. Many candidates failed to recognise that $\frac{dV}{dt} \propto \frac{1}{\sqrt{h}}$ and hence $\frac{dV}{dt} = \frac{k}{\sqrt{h}}$. Of those who had some concept of the rate being $\frac{dV}{dt}$, many misunderstood the idea of inverse proportion and others missed the constant of proportionality. Even where candidates had correct expressions for both $\frac{dV}{dh}$ and $\frac{dV}{dt}$, they often failed to use the chain rule correctly to find $\frac{dh}{dt}$. Many candidates who did link their expressions correctly, achieving $\frac{dh}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$ or equivalent, stated that this could be expressed as $\frac{\lambda}{\sqrt{h}}$, with $\lambda = \frac{k}{200}$ and so gained all three marks. Others made no reference to λ and so lost the accuracy mark. A less common, but correct, approach used $\frac{dV}{dt} = \frac{d(200h)}{dt}$.

Approaches to part (b) of the question were very mixed. Many candidates failed to realise that, as the question said, 'use the model to find an equation', they were expected to, for example, separate the variables and use calculus to solve the differential equation. As they were asked to give their answer in a specific form, a large number started from this and found the constants A and B using the given values of t and h. Fortunately, they were able to gain two marks from the special case. Candidates who used calculus rarely had problems integrating $\sqrt{h}$, although errors were seen later when substituting into $h^{\frac{3}{2}}$ or in dealing with the $\frac{2}{3}$ when rearranging the equation. Candidates who started this part with the λ were generally more successful. Some candidates correctly found c and λ, and then rearranged. Others rearranged before making substitutions, stating $\frac{3}{2}\lambda = A$ and $\frac{3}{2}c = B$. A minority lost the second accuracy mark as they did not give the final equation in the correct form. Some candidates who had correctly found $\frac{dh}{dt} = \frac{k}{200} \times \frac{1}{\sqrt{h}}$ solved the differential equation using $\frac{k}{200}$ in place of λ, in most cases successfully, although the algebraic manipulation was more tricky. Some candidates who had made an incorrect attempt at (a) continued with their answer, usually $\frac{dh}{dt} = \frac{1}{200} \times \frac{1}{\sqrt{h}}$, in this part. As they only had one constant, the 'c', in their expression, they could only attain the first two method marks. It was rare to see the constant of integration missed and basic

Q11
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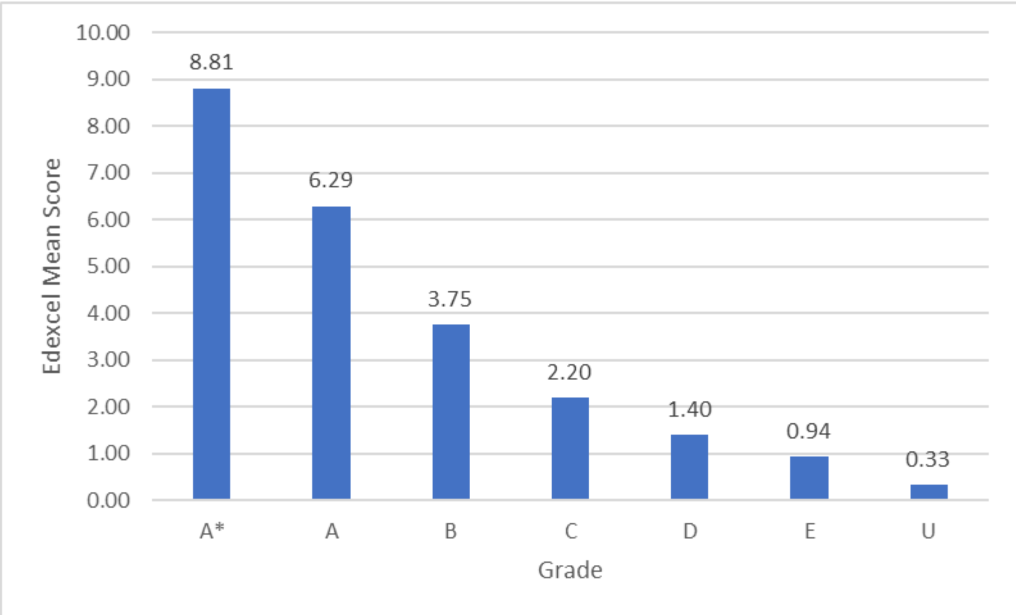
A
B
C

arithmetical errors when finding either λ and c or A and B were also relatively infrequent. The alternative approach, rearranging to give $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{\sqrt{h}}{\lambda}$ or $\frac{200\sqrt{h}}{k}$ was seen on a few occasions.

The majority of candidates who had a correct form of the equation in part (b) made a good attempt at part (c) and were generally able to gain the method mark. Relatively few lost the accuracy mark due to missing the units in their final answer. A number of candidates used an incorrect value for h, most commonly 1000, and so lost both marks.

Question 11 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
4.31	10	43%	8.81	6.29	3.75	2.20	1.40	0.94	0.33



Q11
A
B
C

Question 11 - Response A

a) $\frac{dV}{dt} = \frac{1}{\sqrt{h}}$ $\qquad \frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

~~vol = 1000-h~~

~~$\frac{dv}{dh} = 1000 \quad \frac{dh}{dv} = 1000$~~

vol $= 1000 - 200h$

$\frac{dv}{dh} = -200$

$\frac{1}{\sqrt{h}} \times \frac{1}{-200} = \frac{1}{-200\sqrt{h}}$

$\lambda = \cancel{k}200$

b) when $t=0$, $h = 1.44$

when $t = 8$, $h = 3.24$

$\frac{dh}{dt} = \frac{k200}{\sqrt{h}}$

$\sqrt{h}\, dh = k200\, dt$

$\int h^{\frac{1}{2}}\, dh = \int k200\, dt$

$\frac{2}{3} h^{\frac{3}{2}} = k200t + C$

$\frac{2}{3}(1.44)^{\frac{3}{2}} = 0 + C \quad \therefore C = \frac{144}{125}$

$\frac{2}{3} h^{\frac{3}{2}} = k200t + \frac{144}{125}$

$h^{\frac{3}{2}} = k300t + \frac{216}{125}$

c) $1000^{\frac{3}{2}} = k300t + \frac{216}{125}$

$1000^{\frac{3}{2}} - \frac{216}{125} = k300t$

$31621 = 300t \qquad t = 105.403$ secs.

2/10 marks

Part (a)

B0: $\frac{dV}{dh} = 200$ is not seen.

M0: This candidate attempts to use the correct chain rule but their $\frac{dV}{dt}$ and their $\frac{dV}{dh}$ are not of the required form.

A0*: Follows M0

Part (b)

Q11

A

B

C

M1: Finds the reciprocal and integrates to the required form. Note that this mark was allowed even with no λ in their equation.
A0: Requires λ on the right-hand side.
dM1: Uses $h = 1.44$ with $t = 0$ and attempts to find either c
ddM0A0: No further marks are available for candidates who do not have a λ in their equation.

Part (c)
M0: Substitutes $h = 1000$ not $h = 5$
A0: Follows M0

Examiner comments
Note that this candidate does not solve the given equation in part (b) as λ is assumed to be 200. In this case, only the first 2 method marks are available in part (b).

Question 11 - Response B

a) $\frac{\delta V}{\delta t} \propto \sqrt{h}$: $\frac{dV}{dt} = \lambda\sqrt{h}$

$V = 200h$

$\frac{dV}{dh} = 200$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \lambda\sqrt{h} \times \frac{1}{200}$

$\frac{dh}{dV} = \frac{1}{200}$

$\frac{dh}{dV} = \frac{\lambda\sqrt{h}}{200}$

b) $\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$: $\frac{dt}{dh} = \frac{\sqrt{h}}{\lambda} \rightarrow \frac{1}{\lambda}h^{\frac{1}{2}}$

$t = \frac{2}{3\lambda}h^{\frac{3}{2}} + c$

$\frac{1}{\lambda} \times \frac{2}{3} = \frac{2}{3\lambda}$

$t = 0,\ h = 1.44$

$0 = \frac{2}{3\lambda} \times 1.44^{\frac{3}{2}} + c$

$c = \frac{2}{3}\lambda \times 1.728$

$c = \frac{1.152}{\lambda}$

$c = \frac{1.152}{0.63}$

$c = \frac{64}{35}$

$t = \frac{2}{3 \times 0.63}h^{\frac{3}{2}} + \frac{64}{35}$

$t = \frac{200}{189}h^{\frac{3}{2}} + \frac{64}{35}$

$t = 8,\ h = 3.24$

$8 = \frac{2}{3\lambda} \times 3.24^{\frac{3}{2}} + c$

$8 = \frac{2}{3\lambda} \times 5.832 + \frac{1.152}{\lambda}$

$8 = \frac{3.888}{\lambda} + \frac{1.152}{\lambda}$

$8 = \frac{5.04}{\lambda}$

$8\lambda = 5.04$

$\lambda = 0.63 = \frac{63}{100}$

$\boxed{h^{\frac{3}{2}} = \frac{189}{200}t - \frac{216}{125}}$

$A = \frac{189}{200}$

$B = -\frac{216}{125}$

Q11
A
B
C

c) $5 \times 20 \times 10 = 1000 = V$ $\quad h^{\frac{3}{2}} = \frac{189}{200}t - \frac{216}{125}$

$h = 5$

$5^{\frac{3}{2}} = \frac{189}{200}t - \frac{216}{125}$

$12.90833989 = \frac{189}{200}t$

$t = 13.6596$

$t = 13.7$ minutes.

6/10 marks

Part (a)

B1: For $\frac{\mathrm{d}V}{\mathrm{d}h} = 200$

M0: This candidate attempts to use the correct chain rule but their $\frac{\mathrm{d}V}{\mathrm{d}t}$ is incorrect.

A0*: Follows M0

Part (b) Alternative method

M1: Finds the reciprocal and integrates to the required form.

A1: Correct integration (+c not required)

dM1: Uses $h = 1.44$ with $t = 0$ and $h = 3.24$ with $h = 8$ and attempts to find either c or λ

ddM1: Complete attempt to find c and λ

A0: Incorrect equation

Part (c)

M1: Attempts to substitute $h = 5$ into their equation and proceeds to a value for t.

A0: Incorrect (and must follow a correct equation in (b))

Question 11 - Response C

a) $V = 200h$

$\frac{dV}{dh} = 200$ $\quad \frac{dV}{dt} \propto \frac{1}{\sqrt{h}}$

$\frac{dV}{dt} = \frac{k}{\sqrt{h}}$

$\frac{dh}{dt} = \frac{1}{\frac{dV}{dh}} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}}$

$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$

where $\lambda = \frac{k}{200}$

Question 11 continued

b) $\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$

$\sqrt{h}\, dh = \lambda\, dt$

$\int \sqrt{h}\, dh = \int \lambda\, dt$

$\frac{2}{3}h^{\frac{3}{2}} = \lambda t + c$

when $t = 0$, $h = 1.44$

$\frac{2}{3}(1.44)^{\frac{3}{2}} = (0)\lambda + c$

$\frac{144}{125} = c$

when $t = 8$, $h = 3.24$

$\frac{2}{3}(3.24)^{\frac{3}{2}} = 8\lambda + \frac{144}{125}$

$\frac{486}{125} - \frac{144}{125} = 8\lambda$

$\frac{171}{500} = \lambda$

$\frac{2}{3}h^{\frac{3}{2}} = \frac{171}{500}t + \frac{144}{125}$

$h^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$ ①

$A = \frac{513}{1000}$, $B = \frac{216}{125}$

Q11

?

A

B

C

c) Max $V = 20 \times 10 \times 5 = 1000\,m^3$

sub $h = 5m$ into ①

$$(5)^{\frac{3}{2}} = \frac{513}{1000}t + \frac{216}{125}$$

$$\frac{5\sqrt{5} - \left(\frac{216}{125}\right)}{\left(\frac{513}{1000}\right)} = \cancel{\frac{513}{1000}}t = 18.4 \text{ minutes}$$

10/10 marks

Part (a)

B1: For $\frac{dV}{dh} = 200$

M1: This candidate attempts to use the correct chain rule with all conditions in the mark scheme satisfied.

A1*: Correct proof.

Part (b)

M1: Separates the variables and integrates to the required form

A1: Correct integration (+c not required)

dM1: Uses $h = 1.44$ with $t = 0$ to find a value for their c

ddM1: Uses $h = 3.24$ with $h = 8$ with their value for c and attempts to find λ

A1: Correct equation

Part (c)

M1: Attempts to substitute $h = 5$ into their equation and proceeds to a value for t.

A1: Correct value including units.

Q11

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A

B

C

Question 12

Question 12 - Question

12.

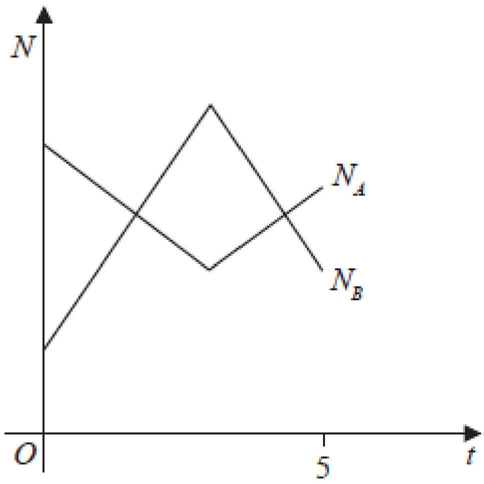


Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A , in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geq 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B , in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geq 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (*c*) and (d).

(*a*) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(*b*) Suggest a value for T, giving a reason for your answer.
(2)

(*c*) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.
(5)

(*d*) State a limitation of the model used for **company B**.
(1)

(Total for Question 12 is 10 marks)

Question 12 - Mark Scheme

Question	Scheme	Marks	AOs
12(a)	$N_A - N_B = (3+4)-(8-6) = ...$	M1	3.4
	5000 (subscribers)	A1	3.2a
		(2)	
(b)	$(T=)3$	B1	3.4
	This was the point when company A had the lowest number of subscribers	B1	2.4
		(2)	
(c)			
	$-t+7=2t+2$ o.e. or $t+1=14-2t$ o.e.	B1	3.1a
	$-t+7=2t+2$ o.e. $\Rightarrow t=...$ or $t+1=14-2t$ o.e. $\Rightarrow t=...$	M1	3.4
	One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	A1	1.1b
	Chooses the outside region for their **two** values of t Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	A1ft	2.2a
	$\left\{t\in\mathbb{R}: t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}: t>\frac{13}{3}\right\}$	A1	2.5
		(5)	
(d)	The number of subscribers will become negative (when $t>7$)	B1	3.5b
		(1)	
		(10 marks)	

Q12 ? A B C

Question 12 - Examiner Comments

In part (a), most candidates were successful in understanding the requirement to substitute zero into N_A and N_B. There were some processing errors resulting in incorrect values for N_A and N_B. A significant number failed to score the method mark by not subtracting their values. There were a few candidates that did not recognise the question was scaled in thousands and left a final answer of 5.

In part (b), a large majority of candidates were able to deduce that $T = 3$ and most explanations were around the idea of the values increasing after this point. The next most common explanation centred on the graph being a minimum or having "turning point" at this location. There were also candidates who stated the gradient has increased or was positive.

In part (c), a significant number of candidates failed to identify either of the correct equations. Many were able to identify and solve correctly at least one equation to find one critical value for t. A common error often followed their correct equation of $5 = 3t$ when they wrote the value of t as 3/5 and similarly, but less common, $13 = 3t$ leading to $t = \frac{3}{13}$. Some students kept the modulus signs within their equations and ignored them at various stages in their working which often led to incorrect equations. There was an array of methods seen trying to negotiate the modulus signs, and $|A|+|B|=|A+B|$ was seen a few times and $|A|+|B|=k \Rightarrow |A|^2+|B|^2=k^2$ was also seen.
Those who found two values for t were generally able to score the 2nd follow through accuracy mark for choosing the outside region for their critical values. The biggest challenge was the final accuracy mark as quite a few candidates did not write the inequalities using set notation. Those who attempted set notation usually were unable to write their answer correctly in that form, some confusing $\cup$ with $\cap$. A common answer was $\left\{t < \frac{5}{3} \cup t > \frac{13}{3}\right\}$ which was condoned for this final mark.

Those who attempted the squaring method in this part had minimal success depending on when they chose to square.

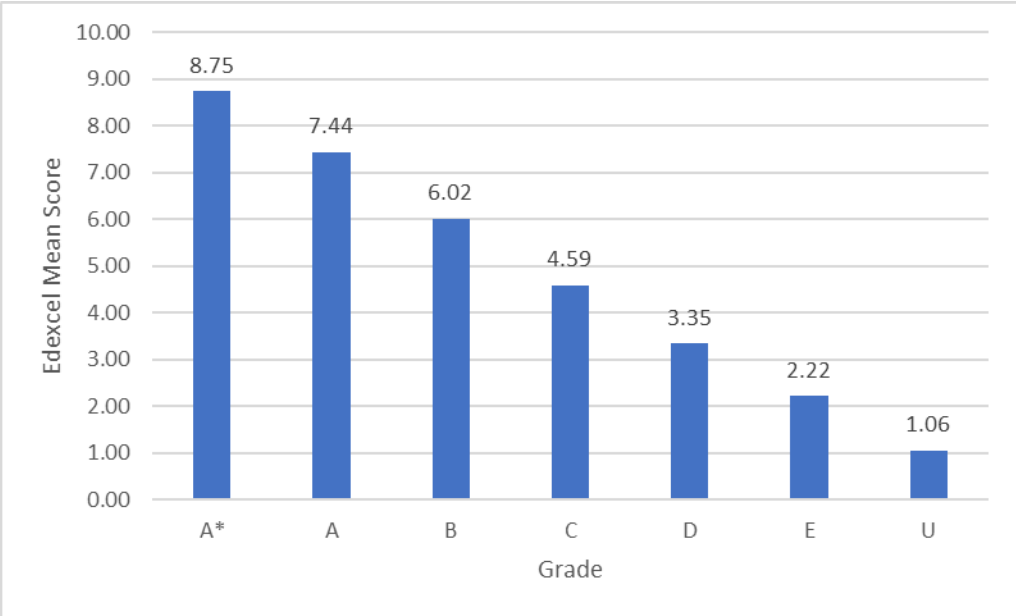
Many candidates were successful in part (d) even if they had not progressed with the other parts of the question. The majority of candidates correctly referenced the subscribers will become negative to obtain this B mark. Where a candidate attempted this part but did not obtain this mark, it was usually due to not making a reference to the number of subscribers becoming negative, saying it goes to zero or that it is linear or decreasing. Several candidates thought it couldn't go above 8000 subscribers.

Q12

A
B
C

Question 12 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
5.76	10	58%	8.75	7.44	6.02	4.59	3.35	2.22	1.06



Question 12 - Response A

12a) $N_A = |0-3|+4$ $\quad N_B = 8-(2\times 0-6|$

$N_A = 7$ $\quad N_B \quad 2$

$= 5000$

b) $Na = |t-3|+4$

$Na = |T-3|+4$

$-4 = |T-3|$

$+4 = T$

$+3$

Q12
A
B
C

c) $|t-3|+4 > \cancel{t8}\ 8-|-2t+6|$

$|t-3|+4 = \cancel{8-|2t-6|}\ 8+2t-6$

$t+3+4 = \cancel{8-2t+6}\ 8+2t-6$

$\cancel{t+7 = 2+2t}$

$\cancel{t+7 = 2-2t}$

$-1 = -5$

$3t = -5$

$\cancel{3t > 5 \div 3}$ $\quad t > 5$

$\cancel{-}|t+3|+4 > 8-2t+6$

$-t+7 > 14-2t$

$\cancel{t > 7}$

$t < 7$

$\cancel{\{t>5 \cup t<7\}}$

$\{t\ Na > Nb : t>5 \cup t<7\}$

c) NB goes into negative if extrapolated,

you can't have negative subscribers

3/10 marks

Part (a)
M1: Finds the difference between the number of subscribers when $t = 0$
A1: Awarded for 5000.

Part (b)
B0: Does not identify $t = 3$
B0: No explanation

Part (c)
B0: Does not form at least one valid equation.
M0: Does not solve at least one valid equation.
A0A0A0: Follows M0

Part (d)
B1: For indicating that the number of subscribers will become negative.

Question 12 - Response B

a) At $t = 0$

$N_A = |0-3| + 4$

$= 3 + 4$

$= 7$

~~A~~ Company A has 7000 ~~subscribers~~. subscribers

$N_B = 8 - |0-6|$

$= 8 - 6$

$= 2$

Company B has 2000 subscribers

Difference $= 7000 - 2000$

$= 5000$ subscribers.

b) $T = 3$ as that is the ~~minimum~~ value of t for which ~~Minimum~~ ~~company A has~~ ~~fewer~~ number ~~of subscribers and the company may have reduced its power to increase and current power~~ company A has its minimum - its lowest number of subscribers.

c) $|t-3| + |2t-6| = 4$

$t^2 - 6t + 9 + 4t^2 - 24t + 36 = 16$

$5t^2 - 30t + 29 = 0$ $\quad \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$t = \dfrac{15 \pm 4\sqrt{5}}{5}$

At $t < \dfrac{15-4\sqrt{5}}{5}$ and $t > \dfrac{15+4\sqrt{5}}{5}$

$\left\{t \in \mathbb{R} : t \to t < \dfrac{15-4\sqrt{5}}{5}\right\} \cap \left\{t \in \mathbb{R} : t \to t > \dfrac{15+4\sqrt{5}}{5}\right\}$

d) Eventually company B will predict a negative number of subscribers, which is not possible, so it is not valid for large ~~number~~ values of t.

5 / 10 marks

Part (a)
M1: Finds the difference between the number of subscribers when $t = 0$
A1: Awarded for 5000.

Part (b)
B1: Identifies $t = 3$
B1: Correct explanation

Q12

?

A
B
C

Part (c)
B0: Does not form at least one valid equation.
M0: Does not solve at least one valid equation.
A0A0A0: Follows M0

Part (d)
B1: For indicating that the number of subscribers will become negative.

Examiner comments
Note that some candidates opted for a squaring approach in part (c) in this question. This was met with varying degrees of success. In this response, the approach is incorrect.

Question 12 - Response C

12a) $t=0, N_A = 7, N_B = 2$.

Difference is 5thousand

b) $(3,4)$

$T=3$, that is the vertex of the modulus graph, number of subscribers increased after this.

c) $-t+7 =$

$N_B = 8-|2t-6|$
$= 8-2t+6$
$= -2t+14$

$N_B = 8-|2t-6|$
$= 8+2t-6$
$= 2+2t$.

$N_A = t+1$

$N_A = 7-t$.

$7-t = 2+2t$.
$5 = 3t$
$\frac{5}{3} = t$.

$t+1 = -2t+14$
$3t = 13$
$t = \frac{13}{3}$.

$N_A > N_B$ when

$\{t: t<\frac{5}{3}\} \cup \{t: t>\frac{13}{3}\}$

d) In the given domain for N_B $(t \geq 0)$ ~~as the t increases, NB will go go negative. Not~~ for large values of t, N_B will become negative (as it continues to decrease past $t=3$).

10/10 marks

Q12 ? A B C

Part (a)
M1: Finds the difference between the number of subscribers when $t = 0$
A1: Awarded for 5 thousand.

Part (b)
B1: Identifies $t = 3$
B1: Correct explanation.

Part (c)
B1: Forms at least one valid equation.
M1: Solves at least one valid equation.
A1: Obtains at least one correct critical value.
A1: Obtains the outside region for their 2 values.
A1: Acceptable answer in set notation.

Part (b)
B1: For indicating that the number of subscribers will become negative.

Question 13

Question 13 - Question

13. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.

(4)

(*b*) Using the answer to part (*a*) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer to 4 significant figures.

(4)

(*c*) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer in the form $a \ln b + c$, where a, b and c are constants to be found.

(5)

(Total for Question 13 is 13 marks)

Question 13 - Mark Scheme

Question	Scheme	Marks	AOs
13(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = 3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2} = 1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2} = \frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	
(b)	$\int 6x\times"\left(\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}\right)"\,dx = \int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = ...$	M1	1.1b
	$\int\left(\frac{2x}{3}-\frac{4x^2}{9}+\frac{2x^3}{9}\right)dx = \frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}$ oe	A1	1.1b
	$\left["\frac{x^2}{3}-\frac{4x^3}{27}+\frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3}-\frac{4(0.4)^3}{27}+\frac{(0.4)^4}{18}\right)-\left(\frac{(0.2)^2}{3}-\frac{4(0.2)^3}{27}+\frac{(0.2)^4}{18}\right)$	dM1	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	A1	1.1b
		(4)	

(c)	Overall problem-solving mark (see notes)	M1	3.1a
	$u=3+x \Rightarrow \int_{3.2}^{3.4} f(u)\,du \Rightarrow \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4} \frac{6}{u}-\frac{18}{u^2}\,du \Rightarrow ...\ln u + ...u^{-1}$	M1	1.1b
	$\int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4} \frac{6}{u}-\frac{18}{u^2}\,du \Rightarrow 6\ln u + 18u^{-1}$	A1	1.1b
	$\left[6\ln u + 18u^{-1}\right]_{3.2}^{3.4} = \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,dx = \frac{...x}{3+x} \pm ...\int (3+x)^{-1}\,dx = \frac{...x}{3+x} \pm ...\ln(3+x)$ oe	M1	1.1b
	$= 6\ln(3+x) - \frac{6x}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) - \frac{6(0.4)}{3+0.4}\right) - \left(6\ln(3+0.2) - \frac{6(0.2)}{3+0.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	M1	3.1a
	$\int 6x(3+x)^{-2}\,dx = \int\left(\frac{...}{(3+x)} + \frac{...}{(3+x)^2}\right)dx = ...\ln(3+x) + \frac{...}{3+x}$ oe	M1	1.1b
	$= 6\ln(3+x) + \frac{18}{3+x}$ oe	A1	1.1b
	$\left(6\ln(3+0.4) + \frac{18}{3+0.4}\right) - \left(6\ln(3+0.2) + \frac{18}{3+0.2}\right) = ...$	ddM1	1.1b
	$6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ oe	A1	2.1
			(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for **all of**

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x} \to b\ln(3+x)$ or $\frac{a}{u} \to b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u = x+3 \to \pm\frac{a}{u} \pm b\ln u$ or e.g. $u = (x+3)^2 \to \pm\frac{a}{\sqrt{u}} \pm b\ln u$
- parts: $\pm a\ln(3+x) \pm \frac{bx}{3+x}$ condone missing brackets e.g. $...\ln x + 3$ for $...\ln(3+x)$
- partial fractions: $\pm a\ln(3+x) \pm \frac{b}{3+x}$ condone missing brackets e.g. $...\ln 3 + x$ for $...\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u = x+3 \to 6\ln u + 18u^{-1}$ or e.g. $u = (x+3)^2 \to 3\ln u + \dfrac{18}{\sqrt{u}}$
- parts: $6\ln(3+x) - \dfrac{6x}{3+x}$
- partial fractions: $6\ln(3+x) + \dfrac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2) + \dfrac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs.
(ignore any reference to a constant of integration)
Do not condone missing brackets e.g. $6\ln x + 3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value
Depends on both previous method marks.
For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.
This could be e.g. $[f(x)]_{0.2}^{0.4} = ...$ provided both previous M marks were scored.
Note that for substitution they may revert back to $3+x$ and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\dfrac{17}{16}\right) - \dfrac{45}{136}$ or exact equivalent e.g. $3\ln\left(\dfrac{289}{256}\right) - \dfrac{45}{136}$ or e.g. $-6\ln\left(\dfrac{16}{17}\right) - \dfrac{45}{136}$

The brackets are not required around the $\dfrac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\dfrac{1}{16}$ but not e.g. $\dfrac{3.4}{3.2}$. The $\dfrac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\dfrac{17}{16}\right| - \dfrac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.

Q13 ? A B C

Question 13 - Examiner Comments

Most candidates attempted at least part (a) of this question, but some found the work more demanding in (b) and (c) and made little progress in either part.

Despite this, there were many fully correct or nearly fully correct attempts at the whole question.

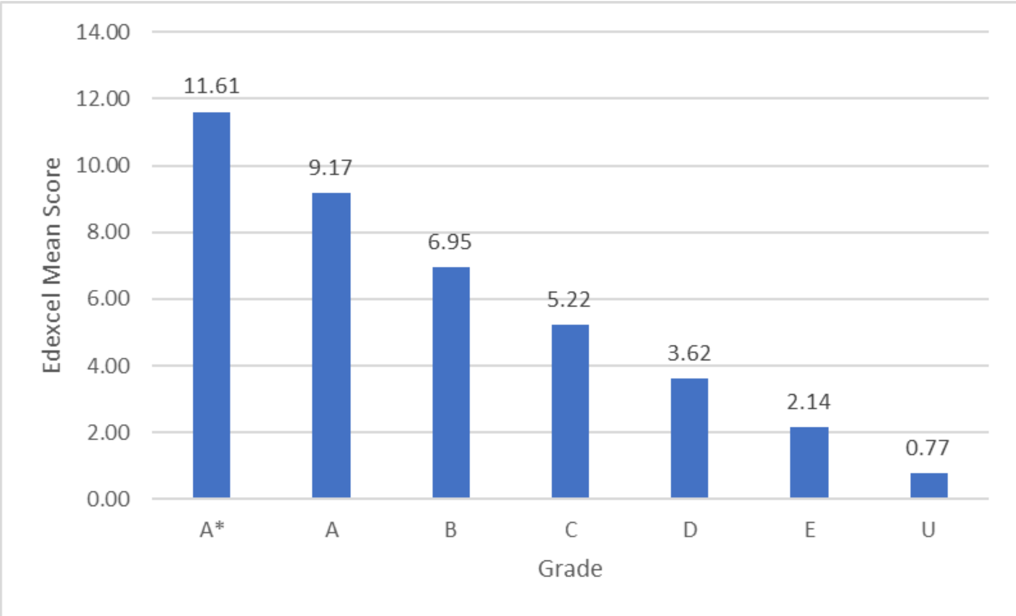
In part (a), most candidates successfully factorised out 3^{-2} and attempted the correct resulting expansion of $\left(1+\dfrac{x}{3}\right)^{-2}$. There were some arithmetic slips, but generally candidates often reached the correct answer. A common error, however, was failing to factorise properly, reaching $3^{-2}(1+x)^{-2}$. There were a number of attempts at direct expansion. Some of these used non-standard binomial coefficients involving negative integers. To achieve credit for using these, candidates needed to demonstrate the meaning of their non-standard notation. Very occasionally candidates multiplied their correct simplified expansion by 9 or 27 at the end. This still received full marks as examiners could ignore subsequent working following seeing a correct simplified answer.

In part (b), many candidates correctly multiplied their expansion by $6x$ and then integrated using the limits of 0.2 and 0.4 reaching 0.03304, gaining full marks. A common error, however, was not giving the final answer to the required accuracy of four significant figures, thus losing the final accuracy mark. Those candidates who had expanded part (a) to more than three terms also lost this final accuracy mark as it led to a different approximation. A more complicated and unnecessary method involving integration by parts was used by a small minority of candidates. Candidates also often tried inappropriate approaches like dividing $6x$ by their expansion followed by a logarithmic integral. A minority of candidates, despite the warning at the start of the question, clearly used calculator technology to obtain 0.032865, rather than the answer of 0.03304 required. Others made inappropriate attempts at the trapezium rule, thus gaining no credit as algebraic integration was required.

In part (c), the commonest methods of integration adopted were substitution, using either $u = x + 3$ or $u = (x + 3)^2$, integration by parts, integration after applying partial fractions and integration using the reverse chain rule after writing the integrand in appropriate form. Candidates had varying success with all of these approaches, gaining a variety of marks. Integration by parts seemed to lead to most arithmetical mistakes. Errors resulted from using parts 'the wrong way round' and others integrated the $(3+x)^{-2}$ term incorrectly. When integrating by substitution some students incorrectly thought that $-\frac{18}{u^2}$ would integrate to a natural logarithm. There was some confusion with the limits; some candidates did not change the limits to "u" values and other who did evaluate new limits for their substitution switched back to an expression in x before substituting their changed limits. When using partial fractions some students struggled with the initial format for the partial fractions and it was not uncommon to see partial fractions of the form $\frac{A}{3+x}+\frac{B}{3+x}$. There was a number of candidates who either did not attempt this part of the question at all or who lacked a correct strategy. There were responses where students attempted to integrate numerator and denominator separately, e.g. $6x$ to $3x^2$ and $\frac{1}{(3+x)^2}$ to a variation of $\ln(3+x)^2$, and proceeded to multiply both results. A number of candidates were awarded just the first mark as their overall problem solving strategy was correct despite the fact they were unable to fully implement it. There were also many candidates who could carry out the work in this part completely successfully.

Question 13 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
6.95	13	53%	11.61	9.17	6.95	5.22	3.62	2.14	0.77



Question 13 - Response A

(5)

a) $3(1+\frac{x}{3})^{-2}$

$3\left(1 + -\frac{2x}{3} + \frac{(-2)(-3)}{2}\left(-\frac{1}{3}x\right)^2\right)$

$+ \frac{6}{2} \times \frac{1}{9}x^2$

$= \frac{1}{3}x^2$

$3\left(1 - \frac{2x}{3} + \frac{1}{3}x^2\right)$

$3 - 2x + x^2$

Q13 ? A B C

b) $$\int_{0.2}^{0.4} \frac{6x}{3-2x+x^2}\,dx$$

$$\int_{0.2}^{0.4} \frac{6x}{3} - \frac{6x}{2x} + \frac{6x}{x^2}\,dx$$

$$\int_{0.2}^{0.4} 2x - 3 + \frac{6}{x}\,dx$$

$$\left[x^2 - 3x + 6\ln x + C \right.$$

$$\left[x^2 - 3x + 6\ln x\right]_{0.2}^{0.4}$$

$$(0.4)^2 - (0.4)3 + 6\ln(0.4) = -5.45774$$

$$(0.2)^2 - (0.2)3 + 6\ln(0.2) = -10.216627\ldots$$

$$-5.45774 - (-10.216627) = 4.759$$

c) $$\int 6x(3+x)^{-2}$$

I DI

$6x$ $\frac{1}{3}$

$$\int (x+3)^{-2}\,dx = \frac{1}{1}\frac{(x+3)^{-1}}{-1} = -1 \times \frac{1}{x+3}$$

$$= \frac{-1}{x+3}$$

Q13

A

B

C

$$\int_{0.2}^{0.4} 6x(3+x)^{-2}\,dx$$

$$(6x)\left(\frac{-1}{x+3}\right) - 6\int -1(x+3)^{-1}\,dx$$

$$\frac{6x}{x+3} - (6)\int \frac{1}{x+3}\,dx$$

$$\left[\frac{6x}{x+3} + 6\ln x+3\right]_{0.2}^{0.4}$$

$$\left(\frac{12}{17} + 6\ln \frac{17}{5}\right) - \left(\frac{3}{8} + 6\ln \frac{16}{5}\right)$$

$$? \quad \frac{45}{136} + 6\ln \frac{17}{16}$$

5/13 marks

Part (a)

M0: They have 3(...) not $\frac{1}{9}$(...)

M1: Correct structure for at least 1 of the required terms

A1: Correct unsimplified expansion of the bracket (note the minus sign in the 3rd term is condoned)

A0: Incorrect

Part (b)

M0: Does not multiply their expansion from (a) by $6x$ (or x) (they divide and then incorrectly split the fractions).

A0dM0A0: Follows M0.

Part (c)

M1: This mark is for the overall approach with all 3 conditions in the mark scheme satisfied.

M1: Integrates to the required form for integration by parts (the missing brackets in the ln term are condoned for this mark)

A0: Incorrect (sign error. Had there been no sign error then the missing brackets would have been recovered on the following line because the substitution would imply them)

ddM1: Correct use of limits.

A0: Incorrect.

Question 13 - Response B

a) $\cancel{3^{-3}}\left(1+\frac{x}{3}\right)^{-2}$

$= \frac{1}{9}\left(1+\frac{x}{3}\right)^{-2}$

$= 3\left(1 + -2\left(\frac{x}{3}\right) + \frac{-2 \times -1}{2}\left(\frac{x}{3}\right)^2\right)$

$= 3\left(1 - \frac{2}{3}x + \frac{1}{9}x^2\right)$

$= 3 - 2x + \frac{1}{3}x^2$

b) $\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\, dx$

$\int_{0.4}^{0.4} 6x\left(3-2x+\frac{1}{3}x^2\right)^{-2} dx$

$u = 6x$ $\qquad \frac{dv}{dx} = \left(3-2x+\frac{1}{3}x^2\right)^{-2}$

$\frac{du}{dx} = \cancel{x}\,6$ $\qquad \cancel{v = -3\left(3-2x+\frac{1}{3}x^2\right)^{-1}}$

Q13
?
A
B
C

c) $\frac{6x}{(3+x)^2} = \frac{A(\cancel{3+x})}{(3+x)} + \frac{B(\cancel{3+x})^2}{(3+x)^2}$

$6x = A(3+x) + B$

$-18 = B$

$x: \quad 6 = A$

$\int \frac{6}{3+x} - \frac{18}{(3+x)^2}\,dx$

$= \left[6\ln 3+x + 6(\cancel{3+x})^{3}\right]_{0.2}^{0.4}$

$6\ln(3+x) + 18(3+x)^{-1}$

$\left(6\ln\frac{17}{5} + \frac{90}{17}\right) - \left(6\ln\frac{16}{5} + \frac{45}{8}\right)$

$6\ln\frac{17}{16} - \frac{45}{136}$

7/13 marks

Part (a)

M1: For $3-2\left(1+...x+...x^2\right)$ which is implied (we condone the fact that the $\frac{1}{9}$ reverts back to a 3)

M1: Correct structure for at least 1 of the required terms (second term)
A0: Incorrect unsimplified expansion of the bracket.
A0: Follows A0.

Part (b)
M0: They do not multiply their expansion from (a) by $6x$ (or x) and attempt to integrate - they have raised their expansion to the power -2 and multiplied that by $6x$ which is incorrect.
A0dM0A0: Follows M0.

Part (c)
M1: This mark is for the overall approach with all 3 conditions in the mark scheme satisfied.
M1: Integrates using partial fractions and reaches the required form for this method.
A1: Correct integration.
ddM1: Uses the correct limits and subtracts.
A1: Correct answer in the required form.

Q13 ? A B C

Question 13 - Response C

(5)

(a) $(3+x)^{-2} = \left[3\left(1+\frac{x}{3}\right)\right]^{-2} = \frac{1}{9}\left(1+\frac{x}{3}\right)^{-2} = \frac{1}{9}\left(1-\frac{2}{3}x+\frac{6}{2}\left(\frac{x}{3}\right)^2+\ldots\right)$

$= \frac{1}{9}\left(1-\frac{2}{3}x+\frac{2x^2}{6}\right)$

$= \frac{1}{9}-\frac{2}{27}x+\frac{x^2}{27}$

(b) $\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx = \int_{0.2}^{0.4} 6x\left(\frac{1}{9}-\frac{2}{27}x+\frac{x^2}{27}\right)dx$

$= \int_{0.2}^{0.4} \frac{2}{3}x - \frac{4x^2}{9} + \frac{2x^3}{9}\,dx$

$= \left[\frac{1}{3}x^2 - \frac{4}{27}x^3 + \frac{1}{18}x^4\right]_{0.2}^{0.4} = \left(\frac{1}{3}(0.4)^2 - \frac{4}{27}(0.4)^3 + \frac{1}{18}(0.4)^4\right)$
$- \left(\frac{1}{3}(0.2)^2 - \frac{4}{27}(0.2)^3 + \frac{1}{18}(0.2)^4\right)$
≈ 0.03304 (4 s.f.)

(c) $\int_{0.2}^{0.4} \frac{6x}{(3+x)^2}\,dx = \int_{3.2}^{3.4} \frac{6(u-3)}{u^2}\,du = \int_{3.2}^{3.4} \frac{6u-18}{u^2}\,du = \int_{3.2}^{3.4} 6u^{-1} - 18u^{-2}\,du$

$u = 3+x$ $\quad$ $x = u-3$

$\frac{du}{dx} = 1$

$du = dx$

x	u
0.4	3.4
0.2	3.2

$= \left[6\ln|u| + 18u^{-1}\right]_{3.2}^{3.4}$

$= \left(6\ln 3.4 + \frac{18}{3.4}\right) - \left(6\ln 3.2 + \frac{18}{3.2}\right)$

$= 6\ln 3.4 - 6\ln 3.2 - \frac{45}{136}$

$= 6\ln\frac{17}{16} - \frac{45}{136}$

13/13 marks

Part (a)

M1: For $\frac{1}{9}\left(1+\ldots x+\ldots x^2\right)$

M1: Correct structure for at least 1 of the required terms.
A1: Correct unsimplified expansion of the bracket.
A1: All correct.

Part (b)
M1: Multiplies their expansion from (a) by $6x$ and attempts to integrate (one power increasing by 1 after multiplying)
A1: Correct integration.
dM1: Uses the limits 0.4 and 0.2 and subtracts.
A1: For awrt 0.03304

Part (c)
M1: This mark is for the overall approach with all 3 conditions in the mark scheme satisfied.
M1: Integrates to the required form for integration by substitution.
A1: Correct integration for this method.
ddM1: Substitutes the correct changed limits 3.4 and 3.2 and subtracts.
A1: Correct answer in the required form.

Question 14

Question 14 - Question

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that the equation

$$2\tan\theta\,(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta\,(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta\,(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(*b*) Hence, solve for $360° \leq x \leq 540°$

$$2\tan x\,(8\cos x + 23\sin^2 x) = 8\sin 2x\,(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

(Total for Question 14 is 7 marks)

Question 14 - Mark Scheme

Question	Scheme	Marks	AOs
14(a)	e.g. $2\underline{\dfrac{\sin\theta}{\cos\theta}}(8\cos\theta+23\left(\underline{1-\cos^2\theta}\right))=8\times\underline{2\sin\theta\cos\theta}\underline{\sec^2\theta}$	B1	1.2
	$2\tan\theta(8\cos\theta+23\sin^2\theta)=8\sin 2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(23\cos^2\theta-8\cos\theta-15)=0$	M1A1	2.1 2.2a
		(3)	
(b)	$\sin 2x(23\cos^2 x-8\cos x-15)=0$		
	$\sin 2x=0\Rightarrow x=360°$ or $540°$	B1	2.2a
	$23\cos^2 x-8\cos x-15\Rightarrow\cos x=-\dfrac{15}{23}$	M1	1.1b
	$\cos x=-\dfrac{15}{23}\Rightarrow x=...$	dM1	1.1b
	$x=360°$, $540°$ and awrt $491°$ only	A1	2.3
		(4)	
			(7 marks)

Question 14 - Examiner Comments

Part (a) was at least started by many but when students failed to make any progress towards the result, they tended not to attempt part (b).

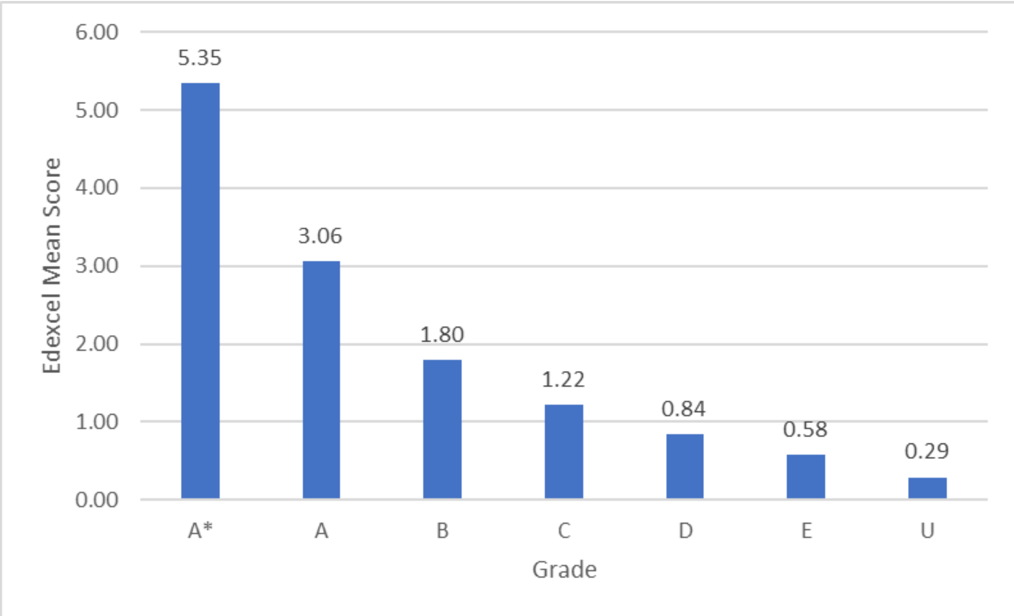
In part (a) the vast majority of candidates scored the first mark for using a correct trigonometric identity. The M and A marks were often both given or both lost, as candidates explored multiple manipulative steps to reach the final form for the equation. In many cases this involved expanding the brackets, using a substitution for sin 2θ, but later putting back into sin 2θ form i.e., undoing the manipulation that had been introduced. Whilst in many cases this resulted in the candidate getting there in the end, there were many examples of multiple attempts or unnecessarily long solutions. The most efficient methods did not involve multiplying out the bracket on the LHS which seemed to be the default first step for most students. A common mistake was to suddenly put the RHS equal to zero so it disappeared rather than being subtracted on the LHS. For those that were successful, there were a variety of approaches as well as variety in the number of steps required to reach the correct final form, with some methods considerably more efficient than others. Arithmetic errors were commonly seen in this question. Many successful candidates attempted to find a factor of sin 2θ on both sides of the equation before proceeding to collecting these on the same side. Another typical successful method involved writing all the terms in terms of sin θ and cos θ, collecting all terms on the same side, and then taking a factor of 2sin θ cos θ. However, many candidates who expanded tended to forget terms when expanding or factorising, leading to an expression in an incorrect format. Replacing tan θ and sin 2θ was usually handled well, as was replacing $1 + \tan^2\theta$ with $\sec^2\theta$ and then $\frac{1}{\cos^2\theta}$ but many candidates didn't follow the manipulation to the end and were unsure what to do with the sin 2θ on the right hand side, so either left it on the right hand side and lost both the M and A marks, or moved it to the left but then incorrectly dealt with it when combined with the quadratic formed on that side.

Part (b) was generally done well by the candidates who managed to obtain a quadratic in part (a), with many candidates correctly solving to get 360 or 540, and then solving their quadratic. Those who found a solution to their quadratic also tended to find solutions based on that within the specified set of values. Where marks were lost, it was often by incorrectly including extra values, in particular the value of 450 degrees which had been specifically excluded in the question.

Q14
?
A
B
C

Question 14 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	**Max score**	**Mean %**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
2.32	7	33%	5.35	3.06	1.80	1.22	0.84	0.58	0.29



Question 14 - Response A

$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1+\tan^2\theta)$

$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$

$1 = \sin^2 + \cos^2$

$\cos \cancel{n} = \sin^2$

$2\tan\theta(8\cos\theta + 23 - 23\cos^2\theta) = 8\sin 2\theta(1+\tan^2\theta)$

1/7 marks

Part (a)
B1: Recalls and uses at least one correct trigonometric identity in the given equation.
M0: Does not manipulate the equation to obtain an expression of the required form.
A0: Follows M0

Part (b)
Not attempted

Question 14 - Response B

$23\sin^2\theta = 23(1-\cos^2\theta)$

$= 23 - 23\cos^2\theta$

$2\tan\theta\left(8\cos\theta + 23 - 23\cos^2\theta\right) = \cancel{8\sin}\ 16\sin\theta\cos\theta \times \sec 2\theta$

$\cos^2\theta \times \quad \frac{2\sin\theta}{\cos\theta}\left(8\cos\theta + 23 - 23\cos^2\theta\right) = 16\sin\theta\cos\theta \times \frac{1}{\cos^2\theta} \times \cos^2\theta$

$2\sin\theta\cos\theta\left(8\cos\theta + 23 - 23\cos^2\theta\right) = 16\sin\theta\cos\theta$

$\sin 2\theta\left(-23\cos^2\theta + 8\cos\theta + 23\right) = 0.$

$A = -23, \quad B = 8, \quad C = 23$

(b) $\sin 2x\left(-23\cos^2 x + 8\cos x + 23\right) = 0$

$\sin 2x = 0$ or $-23\cos^2 x + 8\cos x + 23 = 0$

$2x = 0, 180,$ $\quad \cos x = \cancel{\frac{4+\sqrt{545}}{23}}$ or $\cos x = \frac{4-\sqrt{545}}{23}$

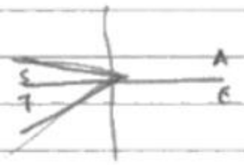
$360, 540,$

$720, 900,$ $\quad x = \cancel{147.26},$

1080 $\quad -1 \le \cos x \le 1$ $\quad \cancel{212.74}$

$x = \underline{360°}, \cancel{450}, \underline{540°}$ $\quad$ not in range. $\underline{507.26}$

$x \ne 450.$



4/7 marks

Part (a)

B1: Recalls and uses at least one correct trigonometric identity in the given equation.

M0: Manipulates their left-hand side expression to obtain an expression of the form $\sin 2\theta\left(A\cos^2\theta + b\cos\theta + C\right)$ but they have not dealt with the right-hand side.

A0: Follows M0.

Part (b)

B1: For x = 360 or 540.

M1: Attempts to solve their quadratic equation in $\cos x$ from part (a) leading to a value for $\cos x$.

dM1: Attempts to find one of their angles in the range (but not 450) for their $\cos x = k$ where $|k| < 1$ which is implied by their value for x.

A0: Follows their incorrect quadratic equation.

Question 14 - Response C

a. $2\tan\theta\,(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta\,(1+\tan^2\theta)$

$\cancel{2\tan\theta}\ \frac{2\sin\theta}{\cos\theta}\,(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta\,\sec^2\theta$

$16\sin\theta + \cancel{23}\ 46\,\frac{\sin^3\theta}{\cos\theta} = 8\sin 2\theta\,\sec^2\theta$

$16\sin\theta\cos^2\theta + 46\sin^3\theta\cos\theta = 8\sin 2\theta$

$8\sin 2\theta\,(\cos\theta) + 23\sin 2\theta\,(\sin^2\theta) = 8\sin 2\theta$

$8\sin 2\theta\,(\cos\theta) + 23\sin 2\theta\,(1-\cos^2\theta) = 8\sin 2\theta$

$8\cos\theta\sin 2\theta + 23\sin 2\theta - 23\cos^2\cancel{\sin 2\theta}\,\theta\sin 2\theta = 8\sin 2\theta$

$8\cos\theta\sin 2\theta + \underbrace{23\sin 2\theta - 8\sin 2\theta}_{15\sin 2\theta} - 23\cos^2\theta\sin 2\theta = 0$

$\sin 2\theta\,(8\cos\theta - 23\cos^2\theta + 15) = 0$

$\sin 2\theta\,(-23\cos^2\theta + 8\cos\theta + 15) = 0$

b. $\sin 2\theta = 0$

$\theta = 360^\circ$ or 540°

not 450

$-23\cos^2\theta + 8\cos\theta + 15 = 0$

$\cos\theta = 1$ or $-\frac{15}{23}$

~~$\cos\theta = 1$~~

$\theta = 360^\circ$ or 490.7°

$x = 360^\circ, 490.7^\circ, 540^\circ$

7/7 marks

Q14

?

A

B

C

Part (a)

B1: Recalls and uses at least one correct trigonometric identity in the given equation.

M1: Manipulates their left-hand side expression to obtain an expression of the form $\sin 2\theta\left(A\cos^2\theta + b\cos\theta + C\right)$ (Only sign and arithmetic slips were condoned for this mark)

A1: Correct equation from correct work.

Part (b)

B1: For x = 360 or 540.

M1: Attempts to solve their quadratic equation in $\cos x$ from part (a) leading to a value for $\cos x$.

dM1: Attempts to find one of their angles in the range (but not 450) for their $\cos x = k$ where $|k| < 1$ which is implied by their value for x.

A1: All angles correct with no extra values.

Q14

?

A
B
C

Question 15

Question 15 - Question

15. A student attempts to answer the following question:

> Given that x is an obtuse angle, use algebra to prove by contradiction that
>
> $$\sin x - \cos x \geqslant 1$$

The student starts the proof with:

> Assume that $\sin x - \cos x < 1$ when x is an obtuse angle
>
> $$\Rightarrow (\sin x - \cos x)^2 < 1$$
>
> $$\Rightarrow \ldots$$

The start of the student's proof is reprinted below.

Complete the proof.

(3)

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Longrightarrow (\sin x - \cos x)^2 < 1$$

(Total for Question 15 is 3 marks)

Question 15 - Mark Scheme

Question	Scheme	Marks	AOs
15	$(\sin x - \cos x)^2 < 1 \Rightarrow \sin^2 x - 2\sin x\cos x + \cos^2 x \ (< 1)$ o.e.	**M1**	1.1b
	Examples: $1 - 2\sin x\cos x < 1,\ 1 - \sin 2x < 1,\ -2\sin x\cos x < 0,\ -\sin 2x < 0$	**A1**	2.2a
	As x is obtuse then $-2\sin x\cos x$ is positive because $\sin x > 0$ and $\cos x < 0$ so we have a contradiction. Therefore $\sin x - \cos x \geqslant 1$ *	**A1***	2.4
			(3 marks)

Q15

?

A

B

C

Question 15 - Examiner Comments

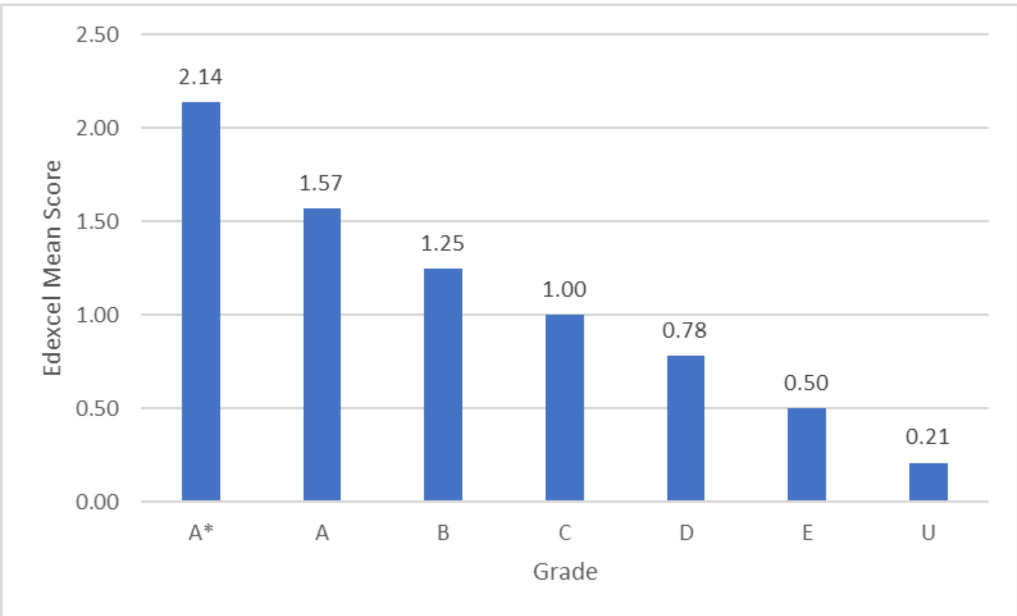
A number of candidates lost all marks on this question by not attempting any solution.

Most candidates correctly expanded the given expression and used $\sin^2 x + \cos^2 x = 1$ to simplify and score the first two marks. A small number incorrectly used $(\sin x - \cos x)^2 = \sin^2 x - \cos^2 x$, scoring no marks. Others attempted to substitute values of 90 and 180 into the given expression instead of using algebraic manipulation.

Whilst most candidates scored the first two marks, only the very best were able to score the final mark, with most candidates failing to offer a convincing reason for a contradiction. Candidates who drew a graph of $\sin x$ and $\cos x$ or $\sin 2x$ to support their understanding tended to score well. Successful candidates typically explained that $2x$ must be between 180 and 360, and hence $\sin 2x$ must be negative, which is a contradiction with their inequality. Those that worked with $\sin x \cos x$ were generally more successful in achieving this mark than those who chose to use $\sin 2x$. However, it was clear that not all students understood what an obtuse angle was. Another typical correct conclusion involved candidates stating if x is obtuse, $\sin x$ must be positive and $\cos x$ must be negative, leading to a negative product and a contradiction. Of those who were able to offer a reason for the contradiction, some still lost the final A mark for failing to then reach a conclusion that the original statement was true or because of errors in the proof such as missing x's or mixed variables.

Question 15 - Performance

			Edexcel averages: mean scored by candidates achieving grade:						
Mean score	Max score	Mean %	A*	A	B	C	D	E	U
1.28	3	43%	2.14	1.57	1.25	1.00	0.78	0.50	0.21



Q15
?
A
B
C

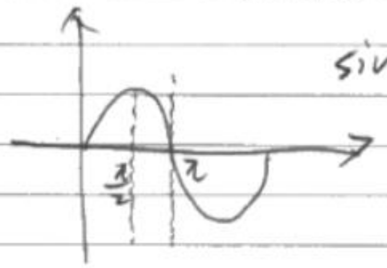
Question 15 - Response A

$\therefore (\sin x - \cos x)^2 < 1$

$\sin^2 x + \cos^2 x - \sin x \cos x < 1$

$\sin x \cos x > 0$

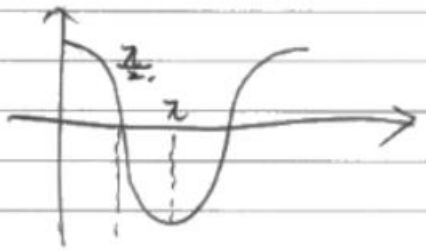
$-\sin x \cos x < 0$



when $x > \frac{\pi}{2}$,

$\sin x > 0$

$\cos x < 0$



$\sin x \cos x < 0$

$\therefore$ ~~it can~~ there is a

1/3 marks

M1: Expands to an allowable form.
A0: Follows an incorrect expansion.
A0*: Follows A0.

Question 15 - Response B

$\sin^2 x + \cos^2 x - 2\sin x\cos x < 1$

$-2\sin x\cos x + 1 < 1$

$-2\sin\cos x < 0$

$-\cos x(2\sin x) < 0$

~~Since x is obtuse, $-\cos x$ is positive and [illegible] ... a positive times positive is always ≥0 ... contradiction that~~

For $-\cos x(2\sin x) < 0$, both $-\cos x$ and $\sin x$ have to be negative or positive

~~Then X is obtuse, since x is positive~~ this only happens when x is acute or between 180°-270°.

Contradiction: x is obtuse not acute or 180°-270°

$\therefore \sin - \cos x \geq 1$ when x is obtuse

2/3 marks

M1: Expands to an allowable form.
A1: Correct inequality after application of the correct identity.
A0*: There are occasions where there are missing variables e.g., line 3.

Question 15 - Response C

$(\sin x - \cos x)(\sin x - \cos x) < 1$

$\sin^2 x - 2\cos x \sin x + \cos^2 x < 1$

$\sin^2 x + \cos^2 x = 1$

$-2\cos x \sin x + 1 < 1$

$-2\cos x \sin x < 0$

$(\div -2)$ as dividing by a negative the sign flips

$\cos x \sin x > 0$

this is a contradiction as when x is obtuse, $\sin x$ is positive and $\cos x$ is < 0

$\therefore$ $\sin x \cos x$ cannot be positive

$\therefore$ By contradiction when x is obtuse, $\sin x - \cos x \geq 1$

3/3 marks

M1: Expands to an allowable form.

A1: Correct inequality after application of the correct identity.

A1*: Convincing argument, states contradiction and conclusion (with no contradictory statements seen). There is a missing *x* but it is only in their side working, not in their proof.

Q15

A

B

C